The Number of Planets, a Number-Referring Term?

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The question whether numbers are objects is a central question in the philosophy of mathematics. Frege made use of a syntactic criterion for objethood: numbers are objects because there are singular terms that stand for them, and not just singular terms in some formal language, but in natural language in particular. In particular, Frege (1884) thought that both noun phrases like *the number of planets* and simple numerals like *eight* as in (1) are singular terms referring to numbers as abstract objects:

(1) The number of planets is eight.

Frege took it as obvious that (1) is an identity statement.

In this paper I will argue that Frege's view about reference to numbers in natural language is fundamentally mistaken. *The number of planets*, I like to show, while it in general is a referential term, is not a term referring to a number (and in fact in the particular context of (1) it is not a referential term at all). In general *the number of planets* does not refer to an abstract object, but rather to what I will call a *number trope*, the concrete instantiation of a 'number property' in a plurality, namely the instantiation of the property of being eight in the plurality of the planets. Moreover, I will argue that (1) is not an identity statement.

1. *The number of planets* as a referential, but not a number-referring term

Let me call terms like *the number of planets* ' *the number of*-terms'. It was Frege's view that since *the number of*-terms are referential terms, they must have the function of standing for an object (Frege's context principle), and since Frege thought that only numbers could be the right objects of reference, numbers are objects. I will argue that in many (though not all) contexts, *the number of planets* has indeed the status of a referential term, but it refers to what I call a 'number trope', a particularized property (or rather relation) which is the instantiation of a 'number property' in a plurality of entities. Thus, *the number of planets* will refer to the instantiation of the number property being eight in the plurality of the planets.

There is a range of semantic evidence that indicates that noun phrases of the sort *the number of planets* (*the number of*-terms for short) do not refer to numbers as abstract objects. First of all, Frege's example (1) cannot be a statement of identity. Substituting the simple numeral *eight* in (1) by an explicit number-referring term results in a sentence that is much less acceptable:[1]

(2) * The number of planets is the number eight.

Even if (2) itself might not convince everyone that (1) is not an identity statement, we will later see linguistic evidence that is rather conclusive to that effect.

But if (1) is not an identity statement, what is its logical form? I will later argue that (1) is neither an identity statement nor a subject-predicate sentence, but rather is of a third sort, namely what linguists call a pseudocleft or specificational sentence, a sentence where (at least on one view) the subject expresses a question and the postcopula NP an answer. *The number of*-terms, however, clearly occur as referential terms in a range of contexts, and I will now focus on those. For example, in contexts such as (3a), *the number of women* satisfies any tests of referentiality. In particular, in that sentence it occurs as subject of a sentence whose predicate generally acts as a predicate of individuals, just as in (3b):

(3) a. The number of women is small.
b. The number eight is small.

Let me call terms like *the number eight* 'explicit number-referring terms'. Explicit number-referring terms and *the number of*-terms display a range of semantic differences with various classes of predicates as well as in other respects. These differences are evidence that the two kinds of terms refer to fundamentally different kinds of entities: *the number of*-terms refer to number tropes; by contrast, explicit number-referring terms refer to abstract objects, to what I will call 'pure numbers'.

2. Predicates

Most importantly, *the number of*-terms and explicit number-referring terms differ in the range of predicates they accept or in the readings they display with particular kinds of predicates.

There are a number of predicates that are perfectly acceptable with *the number of*-terms, but not explicit number-referring terms, and the same predicates, and this is significant, are acceptable also with corresponding plural noun phrases. Such predicates include the comparative predicates *exceed* and *equal*:

(4) a. The number of the women exceeds / equals the number of the men.
b. The women exceed / equal the men in number.
c.* The number fifty exceeds / equals the number forty.

With plural noun phrases, those predicate ask for the modifier *in number*, as in (4b).

One-place predicates of comparative measurement in general display the same pattern with *the number of*-terms, the corresponding plurals, and explicit number-referring terms. Examples of such predicates are *negligible, significant, high,* and *low*:

(5) a. The number of animals is negligible / significant.
b. The animals are negligible in number / significant. (different meaning of the predicate)
c. ?? The number 10 is negligible / significant.

(6) a. The number of deaths is high / low.
b. The deaths are high / low in number.
c. ??? The number ten is high / low.

The closeness of the referents of *the number of*-terms to the associated plurality is also revealed in the readings such terms yield with other evaluative predicates: they do not yield the kinds of readings expected when evaluative and comparative predicates apply to abstract objects, but rather readings that yield an evaluation of the plurality in just one particular respect, namely with respect to how many they are:[2]

(7) a. The number of women is unusual.
b. The number fifty is unusual.

(8) a. John compared the number fifty to the number forty.
b. John compared the number of women to the number of men.

(7a) has quite a different reading from (7b), and (8a) from (8b). In fact, the readings (7a) and (8a) display can be made transparent by the near-equivalence with a sentence just about the plurality such as (9a) and (9b), with a modifier specifying the respect 'in number', that is, the respect of just how many they are:

(9) a. The women are unusual in number.
    b. John compared the women to the men in number.

We can conclude, unlike pure numbers, the entities that *the number of*-terms refer to share certain kinds of properties with the corresponding pluralities. These are precisely the properties that can be attributed to the pluralities when adding the modifier 'in number'. The properties are just the kinds of properties the plurality has when viewed only as 'how many it consist in', that is, when focusing just on how many entities make the plurality up. This gives a first indication of what kinds of entities *the number of*-terms refer to: they are entities that are pluralities 'reduced to' just one of their aspects, namely how many objects they consist in.

There are further properties that show that referents of *the number of*-terms, unlike pure numbers, are entities that are close to the associated plurality. These properties indicate that as long as the plurality consists of concrete entities, the referents of *the number of*-terms also qualify as concrete.

Given common criteria, what defines an entity as concrete rather than abstract is the ability of the entity to act as an object of perception, as an argument of causal relations, and to be spatio-temporally located. It is easy to see that as long as the plurality in question consists of concrete entities, perceptual and causal predicates make sense with *the number of*-terms, though not with explicit number-referring terms:

(10) a. John noticed the number of the women / * the number fifty.
     b. The number of the women / * The number fifty caused Mary consternation.

3. Number tropes

The number of terms, we have seen, refer to entities that have two characteristics:
[1] They share those properties with the corresponding plurality that can be attributed to the plurality with the addition of the modifier 'in number'.
[2] They have properties qualifying them as concrete as long as the corresponding plurality consists of concrete entities.

There is one kind of entity that fits just these two roles, and this is a certain kind of trope, namely what I will call a *number trope*. A trope is a particularized property, a concrete manifestation of a property in an individual (the bearer of the trope). A trope, to put it in another way, is an entity that attends to just one property of a particular individual and disregards all others.[5] A number trope is a trope whose bearer is a plurality and which attends to just the numerical aspect of the plurality, namely just to how many entities the plurality consists in. It disregards all qualitative aspects of those individuals. A number trope is the instantiation of a property being so-and-so-many in a plurality. For example, the trope that *the number of planets* refers to will be the concrete manifestation of the property of being eight in the plurality of the planets.

A number trope differs from standard examples of tropes, such as Socrates' wisdom or the redness of the apple, in that it is purely quantitative. Psychologically speaking, it involves 'abstracting' from all the qualitative respects of a plurality and focussing just on how many it consists in. Ontologically speaking, a number trope is just like the underlying plurality except that it shares only those properties of the plurality that pertain to how many entities it consists in. Other quantitative tropes are John's height, Mary's age, or Bill's weight.

Number tropes have still other kinds of properties than those discussed so far. In particular, number tropes display a range of mathematical properties. But first let us focus on the conception of number tropes itself and the semantics of number trope terms.

The semantics of number trope terms requires an account of plural terms such as *planets*. I take plural terms not to stand for pluralities as single objects of reference, but rather analyse plural terms as referring plurally to various individuals at once (Boolos 1984, Yi 1999, 1995, 2006). On that view, *two* would not be a predicate holding of single objects, plural entities of some sort, but rather it would be a predicate applicable to several individuals at once, and it would be true of several individuals just in case among them are two distinct individuals with which all the others are identical, as in (13):

(11) $\mathit{two}(ww) = 1$ iff $\exists x \exists y(x \leq ww \;\&\; y \leq ww \;\&\; x \neq y \rightarrow \forall z(z \leq ww \rightarrow z = x \vee z = y))$

In (11), 'ww' is a plural variable, that is, a variable that can stand for more than one individual at once , ≤ is the relation 'is one of', and 'x' and 'y' are singular variables, variables that can stand for only single individuals.

Number trope terms are formed with the unspecific functional relational noun *number*. *Number* in trope-referring terms expresses a plural function, a function which maps *n* individuals simultaneously to the trope that is the instantiation of the property of being *n* in those individuals:

(12) For entities dd, *number*(dd) = f(P, dd)) for some number property P such that P(dd).

Here 'f' stands for the function mapping a property or a relation and an individual or several individuals to the instantiation of the property or relation in the individual or the individuals (in case the individual(s) instantiate(s) the property or stand in the relation; it will be undefined otherwise).

The number of-terms may also refer to what appears to be an entity that has variable manifestations as number tropes -- namely at different times or in different possible circumstances

(13) a. The number of students has increased.
b. The number of students might have been higher than it is.

The number of students in (13a) and (13b) does not refer to a single number trope, but rather to a function-like entity, characterized by a function f mapping a world w and a time i to a manifestations that is a number trope in w at i.

One potential semantic problem with number trope terms in English is that *the number of* is actually not followed by a standard plural term, that is, a definite plural NP, but rather by a bare (that is, determinerless) plural. While there are different views about the semantic function of bare plurals, it is generally agreed that bare plurals can act as kind-referring terms (Carlson 1977). In other work, I argue that bare plurals and mass nouns should themselves be considered plurally referring terms, referring plurally to the various instances in the various possible circumstances, so that *rare* would be a plural predicate. In certain contexts, such as that of the functor *the number of*, only the instances of an actualized kind are taken into account, that is, the instances of the kind when restricted to the actual circumstances, that is, the same entities that a definite plural term refers to. Note that in some languages 'the number of' can be followed by a definite plural only (or a specific indefinite), for example in German (*die Anzahl der Planeten / * von Planeten* 'the number of the planets / of planets)'.

The semantics of *the number of planets* is then as follows where $[]^{w,\,i}$ is the restriction of the plurally referring term *planets* to the actual circumstances, the actual world w and the present time i:

(14) [*the number of planets*]$^{w,\,i}$ = f(P, [*the planets*]$^{w,\,i}$), for some number property P that holds of [*planets*]$^{w,\,i}$

Note that on this view *the number of* is not a functor applying to a concept-denoting expression, as Frege assumed. In fact, a concept-denoting expression, a predicate, is impossible in that context (**the number of is a planet*, * *the number of a planet*).

There is one potential problem for the number trope analysis of *the number of* terms and that is cases when the relevant plurality is empty, as in (15):

(15) The number of students this year is zero.

But (15) is in fact a specificational sentence. That is, the subject here has the function of specifying the question ‘How many students are there?’ and the numeral in postcopula position that of giving an answer (see Section 1.3.)

Another apparent problem are identity statements as in (16):

(16) The number of women is the same as the number of men.

There is good evidence, however, that the expression *the same as* in (16) expresses not numerical identity, but rather qualitative identity or close similarity among tropes. This is what *the same as* also expresses with other trope-referring terms:

(18) a. John’s excitement today is the same as John’s excitement yesterday.
b. John’s irritation is the same as Mary’s.

Also identity statements such as (19a), which would have to express numerical identity (or perhaps ‘relative identity’), are bad. Such statements are of course fine with ordinary descriptions as in (19b):

(19) a. ?? The number of women and the number of men are the same number.
b. The carpenter and the professor are the same person.

4. Mathematical properties of number tropes

Number tropes have not only the kinds of properties that are characteristic of tropes in general. They also have certain kinds of mathematical properties. Though they do not share the full range of mathematical properties that pure numbers can have, that is, the referents of explicit number-referring terms. I will argue that the more limited range of properties that number tropes may have (in contrast to pure numbers) follows from the nature of number tropes, as tropes that have concrete pluralities as bearers.

Let us first look at predicates that classify numbers according to their mathematical properties. Predicates such as *even, uneven, finite* and *infinite* are possible both with number tropes and with pure numbers:

(20) a. Mary was puzzled by the uneven / even number of guests.
b. Given the only finite number of possibilities,...
c. John pointed out the infinite number of possibilities.

There are other predicates, however, that are acceptable only with pure numbers but not number tropes. They include *natural, rational*, and *real*:

(21) * the natural / rational / real number of women

Furthermore, many mathematical operations are inapplicable to number tropes. These include one-place operations such as the successorbvbbvv function:

(22) * the successor of the number of planets

By contrast, the two-place functions *sum* and *plus* are applicable to number tropes:

(23) a. the sum of the number of men and the number of women

b. The number of children plus the number of adults is more than a hundred.

What distinguishes the mathematical predicates or functors that are applicable to number tropes from those that are not? The answer to this question can be obtained by reflecting on the kinds of mathematical properties concrete pluralities can have and the kinds of operations that can apply to them.

First of all, there is a sense in which pluralities can be even or uneven: to see whether a plurality is even or uneven, it just needs to be checked whether or not the plurality can be divided into two equal subpluralities. Similarly, in order to see whether a plurality is finite or infinite it simply needs to be seen whether or not a 1-1 mapping can be established from the elements of the plurality onto themselves. A number trope will then be even, uneven, finite, or infinite simply because the plurality that is its bearer is. Let us then state the following generalization: a mathematical predicate is applicable to one or more number tropes just in case its application conditions correspond to hypothetical operations on the pluralities that are the bearers of the number tropes. Such a condition also explains the applicability of the functor *sum*: the sum operation is applicable to two number tropes because it can be defined in terms of an operation on the two pluralities that are the bearers of the number tropes, as in (24):

(24) Addition of Number Tropes

For two number tropes t and t', sum(t, t') = f(P, dd) with P a number property and for individuals ee such that t = f(P_1, ee) and individuals ff such that t' = f(P_2, ff), for number properties P_1 and P_2:

$\forall d(d \leq dd \leftrightarrow d \leq ee \vee d \leq ff)$, provided $\neg\exists d(d \leq ee \,\&\, d \leq ff)$.

Why isn't the successor function applicable to number tropes? The reason is simply that the successor function cannot be viewed as an operation on concrete pluralities: the successor function as a function applying to a concrete plurality would require adding an entity to the plurality. However, given a 'normal' universe, there is not just one single object that could be added, but rather there are many choices as to what object could be added to the plurality to yield its successor. Thus, no uniqueness is guaranteed, which means as an operation on pluralities, the successor function is just not a function. For this reason, the successor function is inapplicable to number tropes. Similar considerations rule out the predecessor, root, and exponent function as operations on number tropes.

Thus we can state the condition arithmetical operations on number tropes as follows:

(25) <u>Condition on arithmetical properties of and functions on number tropes</u>

a. If P is an n-place arithmetical property of number tropes, then for some n-place property of pluralities Q, for any number tropes $t_1, ..., t_n$: $Q(pp_1, ..., pp_n)$ iff $P(t_1, ..., t_n)$ for the bearers $pp_1, ..., pp_n$ of $t_1, ..., t_n$.

b. If f is an n-place function on number tropes, then for some n-place function on pluralities g, for any number tropes $t_1, ..., t_n$: $g(pp_1, ..., pp_n) = f(t_1, ..., t_n)$ for the bearers $pp_1, ..., pp_n$ of $t_1, ..., t_n$.

Again, pp_1, pp_2, .. are plural variables standing for several objects at once.

What about the predicates *natural*, *rational*, and *real*? These are technical predicates that already at the outset are defined just for the domain of all numbers, rather than only the natural numbers. They will therefore not be applicable to number tropes, which are outside the domain of their application.

The possibility of some mathematical properties and functions being applicable to number tropes on the basis of operations on concrete pluralities is also reflected in the acceptability of descriptions of agent-related mathematical operations on number tropes:

(26) a. John added the number of children to the number of adults, and found there were too many people to fit into the bus.

b. John subtracted the number of children from the number of invited guests.

Addition as a mathematical operation performed by an agent, as in (26a), is possible with number tropes for the same reason as addition as a mathematical function. What matters is that the operation as an operation on number tropes is definable in terms of an operation on the underlying pluralities. This does not necessarily mean that when John added the number of children to the number of adults he first mentally put together the plurality of children with the plurality of adults and then counted the result. It just means that if he obtained the correct result, he might as well have obtained it by performing an operation on the concrete pluralities first.

Subtraction of a number trope t from a number trope t' as in (27a) is possible just in case the plurality that is the bearer of t' includes the plurality that is the bearer of t:

(27) a. ?? John subtracted the number of planets from the number of invited guests.

There is an available reading, though, of (27a), a reading more naturally available in a case like (27b):

(27) b. John subtracted the number of passports from the number of applicants.

The reason why (27b) is possible is obviously that it presupposes a natural 1-1 association between passports and applicants. Subtraction will then be an operation on pluralities as well: start with the applicants, associate them with their passports and take away the passports together with their associated applicants, and the number of the remaining applicants will the result of the subtraction.

Division of one number trope by another is, as in (28a), is strange too; though it is acceptable when the second (but not the first) term is a numeral, as seen in (28b) and (28c):

(28) a. ?? John divided the number of invited guests by the number of planets.
b. John divided the number of invited guests by two.
c. ?? John divided eighteen by the number of invited guests.

Divide by two is a complex predicate that involves an arithmetical operation definable as an operation on a plurality. By contrast *divide eighteen by* is not such a predicate: eighteen is not associated with a particular plurality that a division could target, and the plurality of a number trope is not something by which it could be divided.

Again, as with subtraction, there are circumstances, under which a sentence like (28a) is acceptable, for example in the circumstances of (29):

(29) John divided the number of invited guests by the number of tables.

(29) is possible, obviously, because there is a concrete point in associating guests with tables. John's mathematical operation in (29) naturally goes along with an operation on the

underlying pluralities, namely an association of each table with different guests, so that if possible the same number of guests is assigned to each table (that is, the guests of a given table can be mapped 1-1 onto the guest of another table). Thus, again, division is possible because it corresponds naturally to an operation on concrete pluralities.

Multiplication with number tropes is available as well, in certain circumstances, as in:

(30) a. John doubled the number of invited guests.
b. Three times the number of children can fit into the bus.

Those examples, crucially, involve number tropes both as a point of departure and as the result of the multiplication. In (30a), John's act of 'doubling' consists not just in a mathematical operation, but in the replacement of one number trope (the number of invited guests at time t) by another (the number of invited guests at t'). In (30a), the doubling of the number trope may consist in adding as many names as there already are on the list of invited guests. Also (30b) does not just describe a mathematical operation of multiplication of the number of children by three, but rather compares the actual number of children to a hypothetical number trope whose bearer consists in a maximal number of children that fit into the bus. (30b), that is, compares the actual number of children to a hypothetical number trope with three times as many children as bearers.

Arithmetical operations thus are possible with number tropes just in case they can be defined as operations (of a simpler or a more complicated sort) on the underlying pluralities. It is then expected that 'mixed operations' involving both number tropes and pure numbers are excluded. This is indeed the case:

(31) a. * John subtracted the number ten from the number of children.
b. * John added the number twenty to the number of children.

Arithmetical operations are possible on number tropes only in sofar as they are derivative of operations on the underlying pluralities, and pure numbers have no pluralities as bearers.

Number tropes can have only those mathematical properties that are derivative of operations on the underlying pluralities. In addition, number tropes have empirical properties tied to the particular nature of their bearers, properties pure numbers do not have. The difference in the range of properties number tropes and pure numbers may have also shows in the way general property-related expressions are understood with number trope terms and explicit number-referring terms. Such expressions include *investigate*, *property*, and *behaviour*:

(32) a. John investigated the number 888.
b. John investigated the number of women.
(33) a. the properties / behaviour of the number 8 (→ mathematical properties)
b. the properties / behaviour of the number of women (→ other properties)

Whereas (32a) can only mean that John investigated the mathematical properties of 888, (32b) implies that John's investigation was also an empirical one regarding the women in question, namely how many women there were. Similarly, whereas (33a) can only refer to the mathematical properties or the mathematical behaviour of a number, (33b) also refers to nonmathematical, empirical properties or behaviour of the plurality of women.

5. Apparent identity statements

Let us now turn to the problem of apparent identity statements like (1), repeated below:

(1) The number of planets is eight.

One sort of evidence that (1) is not an identity statement involving two number-referring terms comes from the unacceptability of the sentences below:

(34) a. * The number of planets is the number eight.
b. * Which number is the number of planets?
c. * The number of planets is the same number as eight.

But there is even more conclusive evidence that (1) is not an identity statement, to which I will come shortly.

One obvious alternative analysis of (1) to that of an identity statement is that of a subject-predicate sentence, with the subject referring to a trope and the numeral acting as a predicate of tropes. Thus, *eight* as a predicate of tropes would be true of a trope t in case t is the instantiation of the eight-relation in some plurality. However, there is one strong argument against the subject-predicate analysis of (1). The argument is a purely syntactic one: subject-predicate sentences generally do not allow for inversion, illustrated in (45) (Heycock/Koch 1990), whereas (1) does, as seen in (1'):

(35) a. John is honest.
 b. * Honest is John.
(1') Eight is the number of planets.

There is a third kind of sentence besides identity statements and subject-predicate sentences for which (1) is a candidate and that is a specificational or pseudocleft sentence (Higgins 1973, Heycock/Krock 1990). A specificational sentence typically involves a wh question or question-like expression in subject position and a not necessarily referential expression in postcopula position. A typical example is (36a), where the subject takes the form of an indirect question and the postcopula expression is a verb phrase, a non-referential expression:

(36) a. What John did is kiss Mary.

One important analysis of specificational sentences takes them to express relations between questions and answers (den Dikken et al. 2000, Schlenker 2003, Romero 2005).[3] The answer may of course consist in the content of a nonreferential expression, with a complete answer being a completion of that expression as a full sentence.

Crucially, specificational sentences allow for inversion:

(36) b. Kiss Mary is what John did.

(36a) illustrates the most important type of a specificational sentence, in which the subject is a wh-phrase and thus arguably an indirect question. However, there are also specificational sentences with a definite NP as subject, such as:

(37) The biggest problem is John.

Here the subject would be a ‘concealed question’, a non-interrogative expression whose meaning, though, is question-like (Grimshaw 1979). In (47), *the biggest problem* would then stand for a question of the sort ‘what is the biggest problem?’.

There is a particularly strong piece of evidence that (1) is in fact a specificational sentence, rather than an identity statements. It comes from the choice of pronouns in the subject position of specificational sentences in German.

Specificational sentences may contain the pronoun *that* or *it* in subject position, pronouns that can be anaphoric to a preceding concealed question (Mikkelsen 2004):

(38) a. The biggest problem is John; *it* is not Bill.
b. What is the biggest problem? *That* certainly is John.

In English *this* and *it* as in (38) can also be used as ordinary pronouns referring to objects. By contrast, German pronouns in the subject position of specificational sentences can only be *das,* ‘that’ or *es* ‘it’, not pronouns inflected for gender, such as *sie* ‘she’. German *die Zahl der Planeten* ‘the number of planets’ is feminine, but the only pronoun that can replace it is *es* (neutral) or (more colloquial) *das*, unlike in ordinary identity statements where feminine pronouns would have to appear:

(39) a. Die Zahl der Planeten ist acht. Frueher dachte man, es waeren neun.
‘The number of planets is eight. Before it was thought that it were (pl) nine’.
b. * Die Zahl der Planeten ist acht. Frueher dachte man, sie waere neun.
‘The number of planets (fem) is eight. Before it was thought that she was nine.’
c. Maria ist nicht Susanne, sie / *es ist Anna.
‘Mary is not Sue, she / * it is Ann.’

This shows that (1) must be a specificational sentence, with its subject being a concealed question. That is, *the number of planets* in (1) will have as its denotation a question or question-like entity of the sort ‘how many planets are there?’.

Conclusion

For Frege, the construction *the number of planets* was not only indicative of the ontological status of numbers as objects. It was also revealing as to the nature of numbers themselves, namely as objects obtained by abstraction from concepts (Hume's Law). In this paper, we have seen that *the number of*-terms are not number-referring terms and moreover are not obtained by a functor applying to a concept-denoting term. Of course, this does not show that Fregean or Neo-Fregean conception of numbers as objects is mistaken as such, but it means that there is no support for the view from natural language.

Notes

[1] This is despite Frege's own claim to the contrary (Frege 1884). The example is equally unacceptable in German. In fact also Frege's other German example below, where the numeral occurs with a definite determiner is unacceptable in my ears:

(1) ?? Die Anzahl der Planeten ist die Acht.
'The number of planets is the eight.'

[2] One might take *the number of women* in (8a) to be a 'concealed fact' (Grimshaw 1979) rather than a term referring to an object. While this might provide an alternative explanation between (8a) and (8b), it is not applicable in general, for example not to the examples in (9). Moreover, it would not account for the properties of concrete objects that the referents of *the number of* terms display, as discussed below.

[3] An alternative analysis takes speciicational sentences to express higher-order equations, for example Jacobson (1994).

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