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Variable Objects and Truthmaking

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1 Introduction

This chapter will focus on a philosophically significant construction whose semantics brings together two important notions in Kit Fine’s philosophy: the notion of truthmaking and the notion of a variable embodiment, or its extension, namely what I call a “variable object.”

The analysis of the construction this paper will develop will be based on an account of clausal complements of intensional verbs that is of more general interest, based on truthmaking and the notion of a cognitive product, such as a promise or a belief, rather than that of a proposition. On that account, the clausal complement of, for example, *promise* will characterize satisfaction situations of the reported promise, and the clausal complement of *believe* will characterize the truthmakers of the reported belief.

Furthermore, the analysis goes along with an account of modals and conditionals based on truthmakers. That is because the construction itself may involve modals and because it is closely related to one involving adnominal conditionals.

The construction in question consists in definite noun phrases with a relative clause containing an intensional verb such as *need*, as below:

- (1) a. [The book John needs to write] must have impact.

I will call such noun phrases *intensional noun phrases*, *INPs* for short. INPs are noun phrases that have the status of referential terms, but stand for objects that certainly belong to the domain of “shallow metaphysics,” as opposed to “foundational metaphysics,” to use Fine’s terms.¹ Shallow metaphysics, on Fine’s view, has as its subject matter the ontology reflected in ordinary judgments or in natural language, whereas foundational metaphysics deals with the questions of what there really is and may provide the terms to which the ontology of shallow metaphysics may be reduced. The states of Fine’s (2017) truthmaker semantics, for example, belong to the domain of shallow metaphysics, as do intentional objects, the nonexistent objects of thought, as

¹ These terms are not as yet used in Fine’s publications, but only in his lecturing. “Naïve metaphysics” and “critical metaphysics” are alternative terms Fine also uses. The view is implicit in Fine’s (1982) work on nonexistent objects as well as in Fine 2001.

in Fine's earlier work (1982). The latter are related to the sorts of entities INPs stand for, yet they are, as we will see, fundamentally different from them.

A range of criteria indicate that INPs are indeed referential terms and as such stand for objects in the ontology that natural language immediately reflects, the domain of shallow metaphysics. For example, INPs can be antecedents of anaphora in a subsequent sentence, as in the continuation of (1a) below:

- (1) b. It must be widely read.

Moreover, INPs can describe the bearers of tropes (or particularized properties), such as the quantitative length trope below:

- (1) c. The length of the application John needs to write is five pages.

This chapter will develop an account according which INPs stand for *variable objects*. More precisely, they stand for entities that have manifestations as "ordinary" objects in different circumstances and perhaps lack manifestations in the actual circumstance. Moreover, the variable objects in question have manifestations in those and just those circumstances that are exact truthmakers of entities such as John's need in (1a). They are variable objects based on entities needs.

The role of truthmakers assigned in the semantics of described by INPs will be part of a more general account of the meaning of clausal complements of intensional verbs based on truthmaking. While truthmakers will be conceived as exact truthmakers, as in Fine's (2012, 2017) recent work on truthmaking, the truthmaking relation will primarily be viewed as a relation between a situation and a cognitive product, an entity of the sort of a need, a claim, a belief, or a desire, rather than a relation between a situation or state and a sentence.

The notion of a variable object is an extension of Fine's (1999) notion of a variable embodiment. Fine's theory of variable embodiments as formulated in Fine (1999) is about entities that may have different manifestations at different times. The term "variable object," as used in this chapter, is meant to apply to entities that have different manifestations as different objects at different times and in different worlds or situations.² Fine's (1999) notion of a variable embodiment was in the first place meant to account for material objects that allow for a replacement of their parts without loss of identity, such as organisms and artifacts (as well as other sorts of objects allowing for change, such as laws). Variable objects of the sort "the paper John needs to write" seem far removed from entities like organisms and artifacts, which are much more obviously part of our commonsense ontology. However, there are good reasons to attribute to the variable objects described by INPs the status of objects as well, in particular in view of the fact that they serve as bearers of tropes and

² The notion of a variable object differs from that of an arbitrary object (Fine 1985), both in its purpose and in its conception. Arbitrary objects were introduced as semantic values of variables and possibly E-type anaphora. By contrast, the notion of a variable object is an extension of the notion of a variable embodiment, which was meant to account for objects permitting change. Unlike variable objects, arbitrary objects are not (or not generally) associated with a function from circumstances to objects, but rather with a function mapping the object itself to ordinary objects as its values.

are subject to ontological conditions shared by variable embodiments of the more familiar sort.

I will first briefly present Fine's theory of variable embodiments, adding conditions on when variable embodiments bear tropes. I then argue in favor of treating INPs as standing for variable embodiments rather than individual concepts, as would be tempting within the more standard semantic (Montagovian) tradition. The main part of the chapter consists in giving a compositional semantic analysis of INPs within an independently motivated semantics of intensional-verb-clausal-complement constructions based on truthmaking and the notion of a cognitive product. In this context, I show that product-dependent variable objects need to be sharply distinguished from intentional objects, the "nonexistent" objects of thought. Finally, I argue that the very same analysis should be carried over to the construction *the gifted mathematician John claims to be*, analyzed by Grosu and Krifka (2007) in terms of individual concepts.

2 Variable Embodiments

2.1 Variable objects and their properties

The notion of a variable embodiment is central notion in Fine's metaphysics and is meant to account for a great variety of "ordinary" objects. A variable embodiment, according to Fine (1999), is an entity that allows for the replacement of constituting matter and thus may have different material manifestations in different circumstances. Organisms and artifacts, in particular, are variable embodiments. They allow for a replacement of constituting matter and thus may have different material manifestations at different times. Variable embodiments are not identical with their constituting matter, but rather are entities associated with a function mapping a time to their material manifestation at the time. Variable embodiments differ from "rigid embodiments," which are entities that do not allow for a replacement of their immediate parts. An example of a rigid embodiment is a token of the word *be*, which has as its immediate parts a token of *b* and a token of *e*, neither of which can be replaced without loss of identity. A slightly more controversial example of a variable embodiment that Fine (1999) suggests is "the water in the river." The water in the river, conceived as a variable embodiment, will have different realizations as different water quantities at different times.

To account for their modal properties, variable embodiments should be associated not just with a function from times to material manifestations, but with a function mapping pairs consisting of a time and a world to material manifestations—or more generally mapping circumstances to material manifestations. Moreover, in view of the construction this chapter focuses on, the notion of a variable embodiment will be generalized to that of a *variable object*, an entity associated with a function from circumstances to material realizations or entities that themselves may be variable embodiments. Within the spirit of the general approach, I will also introduce a further extension of the notion of a variable embodiment, allowing variable objects to be associated with a function from truthmaking situations to entities in those situations. The situations will be situations that satisfy particular conditions

associated with an event or state described in the sentence. The notion of a variable object will thus be connected with a semantics based on truthmaking.

Variable embodiments generally have properties derivatively, on the basis of their manifestations. In particular, a variable embodiment exists at a time in a world just in case it has a manifestation at that time in that world. Moreover, a variable embodiment shares its location at a time in a world with that of its manifestation at the time in that world, provided it has a manifestation at that time in that world. Finally, a variable embodiment “inherits” time- and world-relative properties from its manifestations in the relevant circumstances. These conditions will have to be generalized so that they can also apply to situations, involving only a partial specification of entities with properties. This means that the inheritance of locational and other properties by a variable object in a situation *i* need to be made conditional upon the manifestation of the variable object having the properties or other properties in *i*. Thus, variable objects are subject to the following conditions, which generalize the conditions on variable embodiments of Fine (1999) from entities associated with functions from times to manifestations to entities associated with functions from circumstances *i* (which may consist of times, worlds, pairs of times and worlds, or situations) to manifestations (which themselves may be rigid or variable embodiments or variable objects):

- (2) a. *Existence*: A variable object *o* exists in a circumstance *i* iff *o* has a manifestation in *i*.
- b. *Location*: If a variable object *o* exists in a circumstance *i*, then *o*'s location in *i* is that of its manifestation in *i* if its manifestation in *i* has a location in *i*.
- c. *Property Inheritance 1*: If a variable object *o* exists in a circumstance *i*, then *o* has a (world- or time-relative) property *P* in *i* if *o*'s manifestation in *i* has *P* in *i*.

(2c) does not yet account for all the properties a variable object may have. It only accounts for its “local properties.” In addition to local properties, which are obtained in the way of (2c), variable objects may have “global properties,” that is, properties that they may have on the basis of several of their manifestations at different times, for example properties of change, rise, or increase.

Variable objects moreover may have properties that are not time- or world-relative. A variable object may have a property in a time- and world-independent way in virtue of all its manifestations having that property. This requires a second condition of property inheritance:

- (2) d. *Property Inheritance 2*: A variable object *o* has a property *P* (circumstance-independently) if all of *o*'s manifestations have *P* in all the circumstances in which they exist and are specified for either *P* or not *P*.

A circumstance-independent property can nonetheless be attributed to a variable object relative to a particular circumstance, assuming that having a property circumstance-independently implies having it in all circumstances. There are certain properties that by nature can be borne only circumstance-independently, for example formal properties such as being an object or being self-identical.

Variable objects are associated with a (partial) function from circumstances to manifestations, but they are not identical to such a function. This is why variable objects can bear properties of concrete objects, unlike abstract functions. This is also why variable objects are able to act as bearers of tropes, being individuals (that is, of type *e*) rather than higher-level semantic values of the type of functions (that is of type $\langle s, e \rangle$).

“Ordinary” objects that are variable embodiments do not generally stand in a 1–1 relation to functions from circumstances to manifestations: not any function from circumstances to manifestations corresponds to a variable object, but rather variable embodiments are driven by various conditions, such as those on shape and continuity. Fine’s theory of variable embodiments frequently faces the criticism of being too generous in what objects it allows; the theory as it stands permits any function from circumstances to entities to be the function associated with a variable embodiment.³ Yet natural language appears to permit reference to a significant range of variable objects. The variable objects natural language allows reference are special, though, in that they are rather strictly tied to the compositional semantics of the constructions in question, rather than fulfilling conditions of individuation independently of it. Variable objects thus appear to be part of a particular construction-driven shallow ontology.

Fine himself (p.c.) meant to apply the notion of a variable embodiment to functional NPs, of the sort *the president of the USA*, as this would allow for an account of functional noun phrases of type *e*, while avoiding treating their referents as abstract functions. Fine himself (p.c.) also thought of applying the theory of variable embodiments to NPs with intensional relative clauses such as *the book John needs to write*, which is of course what this chapter has set out to do. “The book John needs to write” as a variable embodiment does not have a manifestation in the actual circumstances, but only in nonfactual circumstances in which John’s needs are fulfilled.

Treating the semantic value of an INP as a variable object immediately accounts for the obligatory presence of a modal, what I will call, following Grosu and Krifka (2007), the *Modal Compatibility Requirement*, MCR for short. The MCR is illustrated in the contrast between (3a) and (3b):

- (3) a. The book John needs to write must/may have a greater impact than the book he has already written
b. ??? The book John needs to write has a greater impact than the book he has already written

The modal in (3a) permits predicating the predicate of a manifestation in a circumstance other than the actual one. Its obligatory presence with variable objects without actual manifestations follows from Property Inheritance 1: it is only in the presence of a modal that the context of evaluation for the predicate can be shifted to nonfactual circumstances in which the variable object has manifestations falling under the predicate in those circumstances.

³ See, for example, Koslicki 2008. However, Fine (p.c.) clarifies that the theory of procedural postulationism, yet to be fully developed, is meant to deal with the problem.

2.2 Variable objects and criteria for objecthood

Let us first look more closely at the motivations of applying the notion of a variable object to the semantics of functional noun phrases. The standard semantic account of such noun phrases in linguistic semantics is to take them to stand for individual concepts, functions from possible worlds and times to entities. Making use of individual concepts, *the president of the USA* in (4a) will stand for a function mapping a world w and time t to the individual that is the president of the USA in w at t . Similarly, *the temperature* in (4b) on that account will stand for a function from times to degrees.

- (4) a. The president of the USA is elected every four years.
b. The temperature has increased.

Individual concepts have become a standard tool in linguistic semantics since Montague 1973, for the analysis of functional NPs and other constructions. Formally, individual concepts are functions from circumstances to individuals. This does not mean that they act as abstract objects in the domain of entities, objects of type e . Individual concepts rather are the semantic values of noun phrases of type $\langle s, e \rangle$ rather than of type e . Noun phrases that stand for individual concepts, that is, that are of type $\langle s, e \rangle$, contribute a semantic value to the overall compositional meaning of the sentence which is not that of an object acting as an argument of a first-order predicate.

The variable-objects approach, by contrast, takes functional noun phrases to be referential noun phrases, that is, to be of type e , without identifying their referent with an abstract function. One obvious advantage of the variable-objects approach over the individual-concepts approach is that it avoids assigning multiple types to noun phrases and predicates. More importantly, there are several criteria for when a noun phrase is referential and thus stands for an object and they apply to functional noun phrases (and INPs).

One of them is the ability of a noun phrase to act as an antecedent of anaphora (assuming that the latter themselves are referential or of type e). Thus, the functional noun phrases in (4a, b) can act as antecedents of the anaphoric pronouns in the sentences below:

- (5) a. He has enormous responsibilities.
b. It won't increase further.

Another criterion for the referentiality of noun phrases is the applicability of predicates expressing object-related properties. Predicates like *be elected* and *increase* clearly express properties of ordinary objects, but they are applicable to functional noun phrases as well, as in (4a,b). This means that on the individual-concepts approach, they would have to be intensional predicates. By contrast, on the variable-objects approach, they would have a single meaning as first-order predicates.

Let us then turn to INPs. On an analysis using individual concepts, *the book John needs to write* would stand for a function mapping a circumstance in which John fulfills his need to a book written by John in that circumstance. On the variable-objects view, it stands for an entity that has manifestations as books John writes in circumstances fulfilling John's needs.

INPs meet the same two criteria for referential noun phrases. First, they generally support anaphora, as mentioned in the introduction:⁴

- (6) The book John needs to write must have impact. It must be widely read.

Second, INPs allow for object-related predicates, predicates that can hardly be understood as predicates of semantic values of a higher type. Examples are *count*, *describe*, and *enumerate*:

- (7) a. John counted the screws that were missing.
b. John described the personnel that the company needed to hire.
c. John enumerated the things that he needed to buy.

These predicates apply to INPs in (7), most plausibly, with the very same meaning they have when applying to ordinary objects, rather than applying with a derivative meaning to a higher-type semantic value, let's say one that keeps track of the values of a function when applied to different circumstances. In fact, in the question-answer example below *count* must apply with the very same meaning to ordinary NPs and INPs:

- (8) a. What did John count?
b. John counted the legs of the chair and the screws that were missing.

Another important criterion for the semantic value of a noun phrase to have the status of an object rather than a higher-level semantic value is that it can act as the bearer of a trope (that is, a particularized property). Tropes are entities in the world, which means that only objects can be bearers of tropes, not higher-level semantic values. We turn to this criterion now.

2.3 Variable objects as bearers of tropes

To show that variable objects can act as bearers of tropes, a number of further remarks about trope-referring terms in natural language are required. Terms of the sort in (9) are generally taken to refer to tropes (that is, particularized properties) (Williams 1953; Strawson 1959; Woltersdorff 1970; Campbell 1990; Lowe 2006; Mertz 1996):

- (9) a. the wisdom of Socrates
b. the softness of the pillow

Thus, (9a) stands for the particular instantiation of wisdom in Socrates, that is, a wisdom trope that has Socrates as its bearer, and (9b) stands for the instantiation of

⁴ The possibility of anaphora with INPs opens up a new option for treating modal subordination, as below:

- (1) John must write a paper. It should be 10 pp. long.

On the standard treatment of modal subordination, *it* picks up the discourse referent introduced by *a paper* in the preceding sentence so that the discourse referent will be in the scope of a modal quantifying over the same worlds as *must* in the preceding sentence. On the alternative, variable-objects account, *it* stands for the variable object "the book John needs to write" and it should serve to access the relevant manifestations of that object. The second option needs to be explored further elsewhere.

softness in the pillow. While Socrates' wisdom may be exactly similar to, say, Plato's wisdom, it cannot be identical to it.

Also the terms below arguably refer to tropes, namely quantitative tropes, instantiations of properties of the sort being so and so tall, so and so long, or so and so many in an individual (Campbell 1990; Moltmann 2009, 2013b):

- (10) a. the height of the building
- b. the length of the paper
- c. the number of planets

The number trope that is "the number of planets" is the instantiation of the property of being eight in the plurality of the planets (Moltmann 2013a, 2013b). As a particularized property, it is not shared by any equally numbered plurality.

Qualitative and quantitative tropes exhibit the very same properties characteristic of tropes. Here are a range of such characteristics that are particularly well-reflected in natural language.

First, tropes are as concrete as their bearers. If a trope has a concrete bearer, it may exhibit properties of concreteness such as being the object of perception (Williams 1953; Campbell 1990; Lowe 2006):

- (11) a. John noticed the simplicity of the dress.
- b. John observed Mary's politeness.
- c. John noticed the small number of women that were present.

Tropes may also act as relata of causal relations (Williams 1953):

- (12) a. The heaviness of the bag made Mary exhausted.
- b. The number of passengers caused the boat to sink.
- c. The weight of the lamp caused the table to break.

There are other properties of concreteness that tropes may exhibit, such as "description-independence." Description-independence consists in that tropes generally have an internal structure "below" the description used to refer to them. This manifests itself, for example, in the applicability of predicates of description (Moltmann 2007):

- (13) a. John described Mary's beauty.

Tropes differ in that respect from states and facts, which strictly match the content of a canonical description and thus do not accept predicates of description and comparison (on a natural reading) (Moltmann 2009, 2013b):

- (13) b. ?? John described (the state of) Mary's being beautiful.

Related to description-independence is the ability of tropes to have a measurable extent, allowing, for example, the application of the predicate *exceed*, which, again, is not applicable, on a natural reading, to states and facts:

- (14) a. Mary's happiness exceeds Bill's.
- b. ??? The fact that Mary likes Bill exceeds the fact that Mary is tall.
- c. ??? The state of Mary's liking Bill exceeds the state of Mary's being tall.

Tropes referred to with the help of predicates, however determinable, unspecific, or quantificational the predicates may be, are always maximally specific, just like events, and unlike states and facts, entities whose nature is “exhausted” by the content of a canonical description (Moltmann 2007, 2013c). Tropes may share properties of quantitative comparison with their bearers, though in the latter case requiring a qualification of respect:

- (15) a. The number of men exceeds the number of women.
b. The men exceed the women in number.

Another important feature of tropes consists in the way they enter similarity relations. Tropes instantiating the same property are similar, and tropes instantiating the same “natural” property are exactly similar. In natural language, exact similarity is expressed by *is the same as* (which does not imply numerical identity):

- (16) a. The quality of this fabric is the same as the quality of that fabric.
b. The impact of John’s book was the same as the impact of Bill’s book.
c. The height of the desk is the same as the height of the lamp.
d. The number of women is the same as the number of men

Only the *is* of identity expresses numerical identity, rendering the sentences below intuitively false:

- (17) a. ?? The quality of this fabric is the quality of that fabric.
b. ?? The impact of John’s book was the impact of Bill’s book.
c. ?? The height of the desk is the height of the lamp.
d. ?? The number of women is the number of men.

The way *is the same as* and the *is* of identity are understood gives a particularly good indication that the terms in question refer to tropes and not abstract objects such as properties, degrees, or numbers.

Given these criteria, it is easy to see that functional noun phrases are able to describe the bearers of tropes, of three different sorts. The first sort is illustrated in (18a), the second in (18b), and the third in (18c):

- (18) a. The level of the temperature, which has increased unusually, caused the wax to melt.
b. The increase of the temperature was caused by the heating system being out of control
c. The responsibility of the president of the USA exceeds that of the vice president.

The predicates in these sentences make clear that such noun phrases indeed involve reference to tropes. In (18a), *the temperature, which has increased unusually* stands for a variable object, but *the level of the temperature* stands for a trope based on the tropes of the manifestations of the temperature, the variable object, at the current circumstance—a *local trope*. In (18b), *the increase of the temperature* stands for a trope based on the tropes of manifestations of the temperature (the variable object) in a series of consecutive circumstances—a *global trope*. In (18c), *the responsibility of*

the president of the USA stands for a circumstance-independent trope with the president of the USA, the variable object, as its bearer.

(18a, b) can be accounted for by modifying the two conditions on property inheritance given earlier in (2c) and (2d). The two conditions on property inheritance tell when a variable object inherits properties from its manifestations. Given trope theory, this requires corresponding conditions on when a variable object is the bearer of a particular trope in virtue of its manifestations being bearers of particular tropes. On a trope-theoretical view, two objects o_1 and o_2 sharing a (fully specific) property translates as o_1 and o_2 being bearers of two tropes t_1 and t_2 that are exactly similar. In trope-theoretic terms, Property Inheritance 1 and Property Inheritance 2 can thus be reformulated follows:

- (19) a. *Trope Inheritance 1*: A variable object o that exists in a circumstance i bears a trope t in i if o 's manifestation in i bears a trope t' in i such that t' is exactly similar to t .⁵
b. *Trope Inheritance 2*: A variable object o bears a trope t (circumstance-independently) if for any circumstance s in which o has a manifestation $f(o)$, $f(o)$ bears a trope t' in an extension of s such that t' is exactly similar to t in s .

Variable objects may also set up variable tropes, rather than acting as bearers of a single trope, as below:

- (20) The number of students at the school has increased.

Increase is a predicate of variable objects and in (20) it applies to a variable trope, which is an entity whose manifestation at a circumstance i is the number of students at the school at i , that is, the number trope that has as its bearer the manifestation of the variable object “the students at the school” at i . The variability of the trope thus is “driven by” the variability of the bearer. A variable trope driven by the variability of its bearer o has as its manifestation in a circumstance i the trope t that has as its bearer the manifestation of o in i . The noun *number* in (20) thus denotes the function mapping a variable object onto a variable trope, as below:

- (21) For a variable object e , $\text{number}(e)$ = the variable trope o such that for any circumstance i in which o has a manifestation $F(o, i)$, $\text{number}_i(F(o, i))$ = the manifestation of o in i .

Also variable objects as described by INPs can act as bearers of tropes, as applying the various criteria for tropes reference shows. Below predicates of perception and causation indicate trope reference—even in the absence of an actual bearer:

- (22) a. John noticed the number of screws that are missing.
b. The number of screws that are missing caused the table to fall apart.
c. Mary was astonished by the length of the paper John needs to write
d. Mary noticed the amount of repair that is required to make the machine work again.

⁵ In Moltmann (2013d), I argue that in such cases the very same trope is inherited, that is, $t = t'$. This result is a trope with multiple bearers. This is of course a nonstandard view about tropes.

Also predicates of similarity and identity apply to the INPs in the same way as they did to ordinary trope-referring NPs.

- (23) a. The number of women in the room is the same as the number of men in the room.
b. ??? The number of women in the room is the number of men in the room.
- (24) a. The number of books Mary wants to write is the same as the number of books Sue wants to write.
b. ??? The number of books Mary wants to write is the number of books Sue wants to write.

Furthermore, predicates of quantitative comparison are applicable just as they were applicable to ordinary trope-referring terms, and they can alternatively apply to the bearers of the tropes, in the presence of a qualification of respect:

- (25) a. The number of people that fit into the bus exceeds the number of the people that fit into the car.
b. The people that fit into the bus exceed the people that fit into the car in number.

Not all apparent trope-referring terms with INPs involve reference to single tropes with variable objects as bearers. Those that impose the MCR do not:

- (26) The impact of the book John needs to write has to be / ??? is greater than the impact of the book he has already written.

Such examples involve reference to variable tropes, driven by the variability of the bearer. Thus, *the impact of the book John wants to write* refers to a variable object whose manifestation in a circumstance *i* is the impact of the book John writes in *i*.

Descriptions of qualitative tropes with INPs are generally subject to the MCR, whereas those describing quantitative tropes may be exempt from it, as (25a) and (1c), repeated below, illustrate:

- (27) The length of the application John needs to write is five pages.

In (25a) and (27) the predicates apply to quantitative tropes whose bearers would be variable objects. I will come to the question why such examples not impose the MCR later (§4).

The ability of variable objects to act as the bearers of tropes is an important argument in favor of the variable-objects approach and against the individual-concepts approach. Individual concepts, semantic values of a higher type than type *e*, can hardly be considered bearers of tropes. Tropes are entities in the world, themselves of type *e*, and thus cannot have entities of a higher type as bearers.

2.4 Problems for a compositional analysis of INPs based on individual concepts

There are also problems for a compositional analysis of INPs based on individual concepts. Let me briefly discuss how (28) may be analyzed as standing for an individual concept, just focusing on the general features and their problems and leaving out any formal detail:

(28) the book John needs to write

One way would be an extension of Grosu and Krifka's (2007) analysis of the related construction *the gifted mathematician that John claims to be*, which I will discuss in §3.5. Their analysis has three features. First, it involves type-lifting of all predicates to predicates of individual concepts and all singular terms (including proper names) to terms standing for individual concepts. Second, it requires treating all intensional verbs as operators quantifying over circumstances (possible worlds and times). Finally, it interprets the head noun *book* in (28) in the syntactic position in which it appears, as the head of the relative-clause construction, rather than in a lower position, inside the relative clause (an option discussed below). Greatly simplifying, this analysis would yield (29a) as the denotation of *book John needs to write*, where $\min(S)$ is the set of minimal functions with respect to a set of functions S , as in (29b):

- (29) a. $\cup \min(\{f \mid \text{book}(f)\} \cap \{f \mid [\text{John needs to write}](f)\})$
 b. For a set of functions S , $\min(S) = \{f \mid f \in S \ \& \ \forall g \in S (g \subseteq f \rightarrow g = f)\}$

The second set mentioned in (29a) would be the set of partial functions mapping a world w compatible with the satisfaction of John's needs to an object John writes in w .

One obvious problem with this analysis is its excessive use of individual concepts, involving a lifting of all predicates and singular terms to the type of individual concepts. While raising singular terms and argument positions of predicates to the type of individual concepts is not as such problematic technically, the move seems too far-reaching given the motivation of just getting the semantics of (29a) right. The construction in (28) should not really be grounds for abandoning the view that names stand for objects and that predicates in natural language are generally predicates of individuals.

A way of avoiding the excessive use of individual concepts would be as follows. In *the book John needs to write*, the noun *book* is interpreted inside the relative clause, as in (29c):

- (29) c. the e [John needs to write [e [book]_N]]

Here the second occurrence of "e" as the empty element in the determiner position of *book* is taken to stand for a variable that is to be restricted by *book*. The denotation of *book John needs to write* will then be as below:

- (29) d. $\min(\{f \mid f \text{ is defined for any world } w \text{ compatible with the satisfaction of John's needs \& write}_w(\text{John}, f(w)) \ \& \ \text{book}_w(f(w))\})$

It is in fact a common syntactic view that relative clauses, or at least some of them, involve movement of the noun from inside the relative clause into the higher position and that this permits an interpretation of the noun with respect to the lower position (either in virtue of reconstruction of the noun into the lower position or in virtue of syntactic movement being in fact copying of an expression in another position).⁶

⁶ See, for example, Bhatt 2002 and Grosu and Landmann 1998.

This analysis obviously would allow the noun *book* to remain a predicate of individuals. However, in its attempt to avoid type-shifting, the analysis does not go very far. Even though it is plausible that the head noun *book* in (28) is interpreted in the lower position, this would not be possible for functional trope nouns such as *impact* as below:

- (30) the impact of the book John needs to write

There is no place inside the relative clause for a noun like *impact* in (30). *Impact* will have to be interpreted in the upper position, which means it will have to denote a function applying to individual concepts.

The analyses in (29a) and (29c) also make a rather problematic philosophical assumption by having to consider all intensional verbs, including attitude verbs, operators quantifying over worlds of evaluation. For attitude verbs, this view is problematic as it implies closure under logical consequence. The more widely accepted view of attitude verbs is that they express two-place relations between agents and propositions (or three-place relations between event, agents, and propositions). Similarly, modal verbs are not universally considered operators quantifying over worlds but may instead be considered primitive operators (modalism).⁷ In any case, the compositional semantics of (28) should not imply highly controversial philosophical views.

The individual-concepts approach faces another problem namely a problem of uniqueness. This problem arises in the very same way for the variable-objects approach if it is based on possible worlds. For that reason, I will discuss it in the next section.

3 Truthmaker Semantics with Cognitive Products

3.1 Variable objects and truthmaking circumstances

The variable object-approach raises the question what the circumstances are that are involved in the variable object described by an INP—just as the individual-concepts approach would raise the question of the circumstances for which the individual concepts are defined. Let us again look at (28), repeated below:

- (31) the book John needs to write

Given standard possible-worlds semantics, what comes to mind first would be that the circumstances are the worlds in which John's needs are satisfied. However, this raises a problem of uniqueness. In a given world in which John's need is satisfied, John may have written more than one book meeting the need.⁸ Uniqueness is guaranteed only when restricting oneself to a situation that exactly satisfies John's needs: in such a situation there will be a unique book John has written. A given world

⁷ See Forbes 1985.

⁸ The problem of uniqueness does not arise for the construction *the gifted mathematician John claims to be* analyzed by Grosu and Krifka (2007) because they consider the copula *be* in that construction to be the *be* of identity, taking two individual concepts as arguments (see §3.5).

in which John’s needs are satisfied may contain several situations satisfying his need, each containing a different book. In order to guarantee uniqueness, entire worlds should be replaced by situations exactly satisfying John’s need.

A satisfaction situation of John’s needs may impose various constraints on the book John writes in it, constraints the speaker in fact need not know about. Not all the books John writes in a world or situation in which his need is satisfied qualify as “the book John needs to write”: John may, for example, need to write about a particular topic and for a particular readership. This means that the complement clause of *need* may give only a partial characterization of the exact need and thus of the sorts of situations exactly satisfying the need.

“The book John needs to write” will thus be a variable object dependent on John’s need. It is associated with a function from situations satisfying John’s need to individuals. More precisely these situations are situations that are exact satisfiers of John’s need: they are wholly relevant for the satisfaction of John’s needs; they are *exact truthmakers* of the conditions making up John’s needs. Variable objects described by INPs thus require the notion of exact truthmaking or exact satisfaction. However, this relation will be a relation between situations and entities like needs, not a relation between situations and sentences, as in Fine’s recent work on truthmaking (2012; forthcoming).⁹

There are further reasons for taking the variable objects denoted by INPs to involve functions from truthmaking situations to objects, and that is the close connection of such NPs to conditionals and modals. Thus, it has been argued that the semantics of conditionals involves situations rather than possible worlds (Kratzer 2007/2014). In fact, Fine (2012) himself argues for a semantics of counterfactual conditionals based on states that act as exact truthmakers of the antecedent. A semantics of conditionals based on situations furthermore requires a semantics of modals based on situations because of the close connection between conditionals and modals. The antecedent of a conditional sets up the circumstances over which a modal quantifiers may quantify that occurs in the main clause, as below:

- (32) a. If John has participated in the race, he may have won it.
 b. If John hadn’t participated in the race, he could not have won it.

Furthermore, there is a construction involving adnominal conditionals that is semantically closely related to INPs. The construction has been discussed by Lasnik (1996) with examples such as those below:

- (33) a. The price if you pay now is predictable, the price if you wait a year is not.
 b. The outcome if John gets his way is sure to be unpleasant for us.

Let me call noun phrases of this sort conditional noun phrases, CNPs for short. CNPs are also subject to the MCR, as the occurrence of the modal of necessity in (33b) indicates. Without the modal, (33b) is hardly acceptable on the same the same reading. The MCR is not expected for (33a) because (33a) does not involve just counterfactual circumstances.

⁹ For the notion of exact truthmaking see also Rodriguez-Pereyra 2005 and Moltmann 2007.

CNPs on a situation-based approach to conditionals will stand for variable objects associated with a function mapping situations making the *if*-clause true to individuals falling under the head noun. They are thus *conditional-dependent variable objects*. Variable objects based on an entity like a need as well as conditional-dependent variable objects are associated with functions mapping exact truthmakers (of an entity like a need or the antecedent of the conditional) onto objects. Modals in the main clause in turn will then quantify over situations acting as those truthmakers.

In (31), the truthmaking circumstances appear to be determined by a particular condition constituting John's need at a time. However, the truthmaking circumstances may also depend on a particular event described by the verb, as in the examples below:

- (34) a. the book John promised he would write
b. the report John asked Bill to write

In (34a), the circumstances involved in the variable object are not determined by a general condition, such as the one constitutive of John's needs, but rather by a particular event of promising (note that John may have promised different books on different occasions). Similarly in (34b), the circumstances depend on a particular event of asking. More precisely, in (34a), the circumstances involved in the variable object are those that satisfy a particular promise, and in (34b) the circumstances are those that satisfy a particular demand.

A promise is not the same as an act or promising and a demand is not the same as an act of demanding. Rather a promise is the "product" of an act of promising and a demand is the product of an act of demanding, in the sense of Twardowski (1911) (see also Moltmann 2013b, forthcoming a).¹⁰ Products, but not acts or states, have truth- or satisfaction-conditions (Twardowski 1911). Products and acts moreover differ in a number of other respects, for example with respect to their part-whole structure, relations of similarity they may enter, and evaluative properties (Moltmann 2013b, 2014). The variable objects that INPs describe, thus, depend on the cognitive product of the event or state described by the intensional verb in question, that is, the implicit event argument of the intensional verb, given Davidsonian event semantics. I will call such variable objects *product-dependent variable objects*. They are variable objects associated with functions mapping all and only the situations exactly satisfying the product of the event or state in question to individuals of the relevant sort. In (34a), the variable object is associated with a function mapping all and only the situations exactly satisfying a particular promise John made to books John writes in those situations.

¹⁰ Beliefs and needs should not generally be considered (cognitive) products. Rather they are part of a more general category of attitudinal and modal objects (Moltmann 2017, 2019). Attitudinal objects comprise state-related entities such as beliefs as well as result-like entities such as conclusions, which cannot be viewed as products of acts. Needs fall under modal objects. In this paper, I use the term 'cognitive product', somewhat incorrectly, for the more general category of attitudinal and modal objects. The product function *product* used in Section 3.3. then is simply to be understood as a relation of associating, for example, a belief state with the corresponding belief.

3.2 Restrictions on product-dependent variable objects

There are restrictions on which intensional verbs may set up variable objects. It appears that not all intensional verbs describe events or states that have satisfaction situations and thus are able to give rise to product-dependent variable objects. *Imagine*, for example, does not, since it does not exhibit the MCR:

- (35) a. ??? The house John imagines that he owns must / would be palatial
b. The house John imagines that he owns is palatial.

(35a) may be acceptable only if it involves an additional effort on the part of the interlocutor of mentally setting up a counterfactual situation in which what John imagined is the case.

Imagine differs in that respect from verbs like *claim* and *believe*, which are able to set up variable objects, giving rise to the MCR:

- (36) a. The book John claims to have written would have / ??? has taken at least two years to write.
b. The treasure John believes to have found would be / ??? is worth several million dollars.

Imagine allows for INPs without imposing the MCR, as in (35b), as long as the predicate describes what is part of the content of the imagination. The INP in (35b) does not describe a variable object, though, but rather an intentional, nonexistent object that is dependent on an intentional act or state (and others coordinated with it) (Moltmann 2015). Such entities differ from (product-dependent) variable objects in that they carry properties as attributed within a particular intentional context rather than inheriting properties from their manifestations in satisfaction situations as in the case of product-based variable objects. This difference manifests itself also in the sorts of properties that can be attributed to intentional objects and variable objects. Variable objects can be attributed properties in virtue of entities in satisfaction situations having those properties. By contrast, intentional objects can be attributed properties only as far as they are ascribed by a particular intentional act coordinated with the act of setting up the intentional object.

The difference between variable objects and intentional objects is particular striking with transitive verbs. INPs with transitive *need* or *promise* impose the MCR, whereas INPs with transitive *imagine*, *describe* and *think about* do not:¹¹

- (37) a. The house John imagines is / ??? must / ??? would be huge.
b. The women John described is / ??? must / ??? would be blond and tall.
c. The woman John thought about is / ??? must / ??? would be blond and tall.
- (38) a. The house John needs must be / ??? is huge.
b. The assistant John needs must be / ??? is fluent in French.

¹¹ See Moltmann 2005, 2008, and 2015 for more on the semantics of intensional transitive verbs such as *need* and *promise* and “intentional” verbs such as *think about*, *describe*, and transitive *imagine*.

What permits *claim*, *believe*, *need*, and *promise* to set up variable objects, but not *imagine*, *think*, *think about*, and *describe*? The difference resides in the truth or satisfaction conditions associated with the cognitive products of the events or states the verbs describe. A claim and a belief clearly have truth conditions and thus they also have truthmakers (situations making the claim or belief true). Beliefs and claims are thus on a par with desires and needs, which have satisfaction conditions and thus also satisfiers (situations satisfying the desire or need). By contrast, an imagination has neither truth conditions nor satisfaction conditions. A description does not have truth conditions (though possibly correctness conditions). Even a thought in fact is intuitively not something that is true or false: *John's thought is false* is hardly as natural as *John's belief is false*. A thought appears closer to a product of acceptance or entertaining than a belief. This means there won't be situations acting as truthmakers or satisfiers of imaginations, descriptions, or thoughts, and thus no variable object can be based on them.

Psychological and illocutionary intensional verbs like *claim*, *want*, and *promise* also allow for INPs without imposing the MRC:

- (39) a. The house John claims he owns has a swimming pool and a balcony
b. The house John wants / promised has a swimming pool and a balcony.

This is because such verbs allow for an interpretation setting up intentional objects rather than variable objects, an option not available for non-psychological verbs like *need* (Moltmann 2015).

The difference between variable objects and intentional objects regarding the MCR is also reflected in statements describing comparisons of tropes of intentional objects as opposed to the variable tropes generated by the variability of variable objects. Thus (40) is not subject to the MCR:

- (40) The originality of the paper John wants to write exceeds the originality of the papers he has so far written.

In (40), the trope “the originality of the paper John wants to write” has as its bearer an intentional object, a nonexistent “object of thought,” rather than a variable object.

3.3 *The role of truthmakers for the semantics of clausal complements*

How do truthmaking circumstances come into play in the semantics of INP so that such constructions can set up the variable objects in question? The circumstances are circumstances exactly satisfying an entity like a belief, a claim, a need, a promise, or a demand, which are the products associated with the act or state described by the verb. Like cognitive acts or states, products are cognitive entities and share the same spatio-temporal location as the corresponding acts or states, but they differ fundamentally in their properties, in particular with respect to their ability to bear truth or satisfaction conditions.

The notion of a cognitive product allows dispensing with the notion of an abstract proposition (Moltmann 2013b, 2014). Products play a different role, though, in the meaning of sentences than propositions. Clausal complements do not stand for products. One reason is that products come with a force. Thus, John's desire that

S differs from John’s belief that S with respect to its force. The force is given by the attitude or modal verb, not the *that*-clause. Another reason why clausal complements do not stand for products is that products may involve a more specific content than the *that*-clause and thus carry the right truth- or satisfaction-conditions of the described attitude or speech act, rather than the clausal complement itself. Thus, as, Fara (2013) has pointed out, in desire reports the clausal complement may underspecify the satisfaction conditions of the reported desire, for example in (41a, b):

- (41) a. Fiona wants to PRO catch a fish.
 b. Charlotte wants to have some champagne.

The desire described by (41a) is not just satisfied in case Fiona catches a fish, but only when she catches a fish suitable for eating let’s say. Similarly, the desire in (41b) is not satisfied if Charlotte drinks an amount of champagne that makes her dizzy or she drinks bad champagne that gives her a headache.

Making use of cognitive products in the semantics of attitude reports allows for a straightforward account of the possible underspecification of the content of the attitude by the clausal complement, namely by taking the clausal complement to give only a partial characterization of the content of the cognitive product in question. A *that*-clause may characterize a desire, claim, or other cognitive product in various ways, by giving necessary conditions on the truth of the described desire or claim, by specifying what the desire or claim is about, or by giving form-related properties. For present purposes, a simplified account will suffice according to which the clausal complement just (partially) characterizes the situations that act as truthmakers of the reported belief or as satisfiers of the reported desire or whatever the cognitive product in question.

The situations in question will thus be exact truthmakers or satisfiers of the cognitive product (such as a belief or desire), but they will be only inexact truthmakers of the embedded clause; that is, the situations may not be fully relevant for the truth of the clausal complement.¹²

Following Fine (2017), I take “ \Vdash ” to stand for the relation of exact truthmaking and “ \Vdash ” for the relation of inexact truthmaking, though “ \Vdash ” and “ \Vdash ” now are used as symbols in the object language used to translate natural language sentences into logical forms. A desire report as in (41a) thus will have the logical form below:

- (42) a. $\exists e(\text{want}(e, \text{Fiona}) \ \& \ \forall i(i \Vdash \text{product}(e) \rightarrow i \Vdash \text{PRO catch a fish}))$

Here the attitude verb is taken to express a relation between events or states and agents and involves universal quantification over situations that are exact truthmakers of the product of the event or state and inexact truthmakers of the complement clause.

¹² This analysis simplifies for present purposes and ultimately need to be replaced. See, for example, Moltmann (2019).

Using a standard situation-based semantics according to which constituents of sentences denote functions from situations to extensions, (42a) will be equivalent to (42b):

$$(42) \quad b. \exists e(\text{want}(e, \text{Fiona}) \ \& \ \forall i(i \Vdash \text{product}(e) \rightarrow \exists x(\text{fish}_i(x) \ \& \ \text{catch}_i(\text{Fiona}, x))))$$

More generally, the compositional semantics of the construction attitude verb-clausal complement will be as below (setting aside the treatment of controlled (infinitival) clauses and the associated issue of attitudes *de se*):¹³

$$(43) \quad a. \text{For an attitude verb } V \text{ and clause } S, [V \text{ that } S] = \lambda e x[V(e, x) \ \& \ [\text{that } S](\text{product}(e))]$$

$$b. [\text{That } S] = \lambda d[\forall i(i \Vdash d \rightarrow i \Vdash S)]$$

Here the *that*-clause *that S* is taken to correspond to a set of situations in which *S* is true, that is, that are inexact truthmakers of *S*.¹⁴

There is further linguistic support for the analysis in (43) according to which clausal complements are predicated of cognitive products. In many languages, there are complex intensional-predicate constructions that involve explicit reference to the product of which the clausal complement is predicated. For example, in English the verb *need* alternates with the verb construction *have a need*; Italian has only the complex predicate for “need” *avere bisogno* and French *avoir besoin*. Harves and Kayne (2012) even argue that the English verb *need* is the result of incorporating the copula *have* and the noun *need*. Given the complex-predicate construction, the cognitive product will be available as part of the compositional semantics of the complex predicate *have+a need*, as in the analysis of (44a) in (44b):

$$(44) \quad a. \text{John has a need [to write a book]}$$

$$b. \exists e[\text{have}(\text{John}, e) \ \& \ \text{need}(e) \ \& \ \forall i(i \Vdash e \rightarrow (\exists d(\text{write}_i(\text{John}, d) \ \& \ \text{book}_i(d))))]$$

This semantics is neutral regarding the lexical semantics of intensional verbs and not based on an analysis of modal, attitude, and speech act verbs as quantifiers ranging over possible worlds. All it assumes is that such verbs specify entities (cognitive products) that have truthmakers or satisfiers and that the complement clause gives a partial characterization of them.

3.4 *The semantics of variable objects based on cognitive products*

We can now turn to the compositional semantic analysis of INPs. The analysis will assume that the head noun in an INP has a syntactic presence inside the relative clause and that it receives its interpretation with respect to that position. That is, (45a) will be interpreted as in (45b):

¹³ For independent motivations of conceiving of *that*-clause-complements of attitude verbs as predicates of the product of the described event or rather the relevant attitudinal object, see Moltmann 2013d, 2017.

¹⁴ By contrast, in Fine’s (2017) truthmaker semantics, the meaning of a sentence consists of a set of exact truthmakers and a set of exact falsmakers.

- (45) a. book that John promised to write
 b. [that John promised to write $[[e]_D$ [book] $_{NP}]_{DP}]_{CP}$

This is necessary since the evaluation of “book” should be relativized to situations satisfying John’s promise. That is, *book* needs to be interpreted inside the scope of *promise*. I will assume that a noun phrase of the sort $[e \text{ book}]$ will be interpreted as a restricted variable, with the empty determiner *e* contributing the variable and *book* contributing the restriction.

The denotation of the relative clause in (45a) will then be, simplified, as in (45c):

- (45) c. $\lambda d[\exists e(\text{promise}(e, \text{John}) \ \& \ \forall i(i \Vdash \text{product}(e) \rightarrow (\text{write}_i(\text{John}, d) \ \& \ \text{book}_i(d))))]$

That is, (45a) denotes the property of being a variable object *d* such that for some promise of John’s, in any satisfaction situation of that promise, *d* is a book John writes.

The property in (45c) will not identify a unique variable object, but holds of all variable objects that have realizations in the satisfaction situations of John’s promise that are (suitable) books John writes. To obtain uniqueness requires recourse to an ordering among variable objects and use of a minimality operator with respect to that ordering. The ordering will be a particular part–whole relation \leq among variable objects, as below, where *F* is the function mapping a variable object onto the function from circumstances to manifestations of the variable object in those circumstances:¹⁵

- (46) For variable objects *d* and *d'*, $d \leq d'$ iff $F(d) \subseteq F(d')$.

(45a) will then stand for the smallest variable object relative to that ordering (\min_{\leq}), as in (47):

- (47) $\min_{\leq} d[\exists e(\text{promise}(e, \text{John}) \ \& \ \forall i(i \Vdash \text{product}(e) \rightarrow \text{write}_i(\text{John}, d) \ \& \ \text{book}_i(d)))]$

Such an analysis can be applied also to relative clauses with modal verbs expressing necessity such as *the paper John must write*. As with *need*, the product in this case will consist of whatever overall condition it is that John must fulfill.

Variable objects can also be set up with relative clauses containing a modal of possibility:

- (48) The book John is allowed to write might have a much greater impact than the one he is not allowed to write.

The same analysis applies here: the book John is allowed to write in (48) involves quantification over the satisfiers of a permission.

¹⁵ It is conceivable that two distinct variable embodiments share the same function from circumstances to manifestations. But this can hardly be so for variable objects of the sort “the book John promised to write.” There is not much more to variable objects of this sort than what is given by the partial function from truthmaking situations to manifestations. Thus, it is reasonable to assume that there is a unique variable object corresponding to the partial function from circumstances to objects.

Variable objects as described by INPs must go along with a suitable account of modals since, as we have seen, they may interact semantically with modals occurring in the main clause, as below:

- (49) a. The book John promised to write must have impact.

Rather than quantifying over possible worlds of a given modal base, the modal in (49a) quantifies over the satisfaction situations of John's promise, the satisfaction situations of the cognitive product on which the variable object depends. Given the present approach to embedded sentences, this means that the modal product depends anaphorically on the promise. The semantics of (49a) will then be as in (49b), which is equivalent to (49c):

- (49) b. $\exists e(\text{must}(e) \ \& \ \forall i(i \Vdash \text{product}(e) \rightarrow \text{have impact}([\textit{the book John promised to write}])))$
 c. $\exists e(\text{must}(e) \ \& \ \forall i(i \Vdash \text{product}(e) \rightarrow \text{have impact}_i(\text{mind}[\exists e'(\text{promise}_i(e', \text{John}) \ \& \ \forall i'(i' \Vdash \text{product}(e') \rightarrow (\text{write}_{i'}(\text{John}, d) \ \& \ \text{book}_{i'}(d)))])))$

In (49c), $\text{product}(e)$ and $\text{product}(e')$ should have the same content and thus the same truthmakers.

3.5 “The gifted mathematician John claims to be”: identificational variable objects

There is a construction closely related to the INPs discussed so far, involving the copula verb *be*:

- (50) The gifted mathematician John claims to be could solve this problem in no time.

This construction, which I will call *identificational INPs*, has been discussed and analyzed in terms of individual concepts by Grosu and Krifka (2007). On Grosu and Krifka's analysis, roughly, the subject of (50) stands for an individual concept mapping any circumstance in which what John claims is true onto a gifted mathematician identical with John in that circumstance.

There are the very same arguments in favor of the alternative analysis in terms of variable objects. On that view, the subject of (50) refers to the variable object that has in any circumstance exactly satisfying John's claim a manifestation that is an individual identical to John and a gifted mathematician in that circumstance. Let me call such variable objects *identificational variable objects*.

Let us first extend the observations reported in Grosu and Krifka (2007) by noting that identificational INPs may also involve the copula verbs *become* and *remain*:¹⁶

¹⁶ This means that *be* in identificational INPs is not the *be* of identity. In fact, *is identical to* is rather bad in that construction:

- (i) ??? The mathematician John claims to be identical to would be able to solve the problem in no time.

- (51) a. The mathematician John wants to become should be able to solve the problem in no time.
b. The honorable person that John should remain would pay back his debts in time.

Identificational INPs share all the relevant characteristics of INPs so far discussed.

First, instead of a truth- or satisfaction-directed attitude verb like *claim*, identificational INPs may involve a modal verb of necessity or possibility, as below:¹⁷

- (52) The painter John could have become might have initiated a new art movement.

Moreover, like other INPs, identificational INPs require the definite determiner *the* and are subject to the MCR (Grosu and Krifka 2007):

- (53) a. ??? A famous painter John could have become might have initiated a new art movement.
b. ??? The gifted mathematician John claims to be has solved the problem in no time.

Furthermore, identificational INPs allow for anaphora support (Grosu and Krifka 2007). Thus, (50) can be continued by (54):

- (54) He would have no difficulty with it at all.

Finally, identificational INPs can specify the bearers of tropes

- (55) The height of the basketball player Joe would like to become be exceeds the height of any basketball player I know.

Examples such as the following will of course involve variable tropes:

- (56) a. The giftedness of the mathematician John claims to be would be extraordinary.
b. The influence of the poet John could have become would by far exceed the influence of the painter John did become.

Thus, there are good reasons to take identificational INPs to stand for variable objects just like other INPs.

There is one particular challenge that identificational INPs pose, though, and that concerns the interpretation of the copula. The head noun of identificational INPs should be interpreted in the predicate position of the copula verb introducing a restricted variable for variable objects, as below:

- (57) the [e [John claims [PRO to be [e mathematician]]

But then the copula verb would have to express the identity of the subject referent with the manifestation of the variable object at the relevant circumstances. Of course,

¹⁷ According to Grosu and Krifka (2007), the construction allows only for modal verbs of necessity, which is not correct.

this is not compatible with the standard view according to which the predicative complement of *be* expresses a property predicated of the subject referent.

There is reason, however, to take indefinite complements of *be* to not just express a property to be attributed to the subject referent, but to introduce an object with which the subject referent is to be identified. There is a significant linguistic difference between indefinite and bare noun complements of copula verbs, as in illustrated below:

- (58) a. John is a mayor.
 b. John is mayor.

(58a) identifies John as a mayor, whereas (58b) merely specifies John’s profession. The difference is reflected also in the sorts of questions to which (58a) and (58b) are answers. (58a) goes along with the proform *what*, as an answer to the question *what is John?* (58b) goes along with the proform *who*, as an answer to the question *who is John?* This indicates that the semantic contribution of the full indefinite complement *a mayor* of a copula verb will be not be the simple property $\lambda x[\text{mayor}(x)]$, but rather the property $\lambda y[\exists x(\text{mayor}(x) \ \& \ y = x)]$. More generally we have:¹⁸

- (59) For a copula verb *V*, a nominal *N'*, and a circumstance *i*,
 $[V \ a \ N']^i = \lambda z[V_i(z, \lambda y[\exists x(N'_i(x) \ \& \ y =_i x)])]$

In (59), the identity symbol is relativized to a circumstance, allowing for the identity of the manifestation of the variable object with the subject referent at the circumstance in question.

Based on the syntactic structure in (60a), *the mathematician John claims to be* can be interpreted as the variable object given in (60b), which is equivalent to (60c):

- (60) a. the [e [John claims [PRO to be [e mathematician]]]
 b. $\min d[\exists e(\text{claim}(e, \text{John}) \ \& \ \forall i(i \Vdash \text{product}(e) \rightarrow be_i(\text{John}, \lambda y[M_i(d) \ \& \ y =_i d])]))]$
 c. $\min d[\exists e(\text{claim}(e, \text{John}) \ \& \ \forall i(i \Vdash \text{product}(e) \rightarrow M_i(d) \ \& \ d =_i \text{John}))]$

Treating “the mathematician John claims to be” as a variable object distinct from John again goes far beyond what most philosophers and ordinary people, upon reflection, may be willing to accept. But it is part of the shallow ontology displayed by natural language—a part of shallow ontology driven entirely by the content of a particular construction.

4 Exemptions from the Modal Compatibility Requirement

Let us finally turn to the difference between variable objects subject to the MCR and variable objects not subject to it. The difference can be explained entirely in

¹⁸ For an account of indefinite complements of copula verbs that goes in that direction and for further arguments for such an account see Beyssade and Sorin 2005.

ontological terms. Let us recall that the MCR does not hold in (61a) and (61b), in contrast to (62a) and (62b):

- (61) a. The number of people that can fit into the bus exceeds the number of people that can fit into the car.
- b. The length of the vacation John is allowed to take exceeds the length of the vacation Mary is allowed to take.

- (62) a. The impact of the book John needs to write ?? exceeds / ok must exceed / ok might exceed the impact of the book he has already written.
- b. The elegance of the dress the bridesmaid should wear ?? does not exceed / ok should not exceed the elegance of the dress that the bride will wear.

At first sight, the generalization seems to be that INPs referring to quantitative tropes are not subject to the MCR. But this is not right. The MCR is in place below:

- (63) The number of people John might invite ?? exceeds / ok might exceed the number of people Mary might invite.

Yet the distinction between quantitative and qualitative tropes does matter. This is illustrated by the difference between (64a) and (64b) with a one-place evaluative predicate:

- (64) a. The number of papers John has to write during this program is too high.
- b. The quality of the paper John must write ?? is very high / ok must be very high.

The exemptions from the Modal Compatibility Requirement follows a general condition on when a variable object is the bearer of a particular sort of trope on the basis of its instances bearing particular tropes. This is the condition Trope Inheritance 2. Variable objects can act as the bearer of a single quantitative trope because quantitative tropes can easily enter relations of exact similarity, more easily so than qualitative tropes. Let us take (61a). It is quite plausible that the same number of people fit into the bus / the car in the various relevant circumstances, or at least that this is how agents generally perceive things. This means that the number tropes in the relevant circumstances are exactly similar, and given Trope Inheritance 2 that the variable object itself will bear an exactly similar number trope. Trope Inheritance 2 allows a variable object to be the bearer of a single trope on the basis of exactly similar tropes of its manifestations.

Exact similarity among qualitative tropes is unlikely to obtain, given that natural language predicates in general do not express natural qualitative properties, but unspecific, determinable ones. Thus, in (62a), for example, it can hardly be the case that the impact of the book John writes in a situation satisfying John's needs is the very same as the impact of the book he writes in any other situation satisfying his needs. Similarly, in (62b) it will hardly be the case that the elegance of the bridesmaid's dress in one situation satisfying the relevant conditions is the very same as the elegance of her dress in any other situation satisfying the relevant condition.

The ontological nature of the exemption from the MCR gives significant further support for the variable-objects account of the semantics of INPs.

5 Conclusion

Here is a summary of the most important points this paper has established.

First, the various linguistic criteria have supported the view that INPs stand for variable objects, as entities of their own and as part of the shallow, construction-driven ontology of natural language. The variable objects that INPs stand for are to an extent subject to the very same ontological conditions as drive variable embodiments in general.

The variable objects that INPs stand for crucially involve situations, rather than entire worlds, namely exact truthmakers of the cognitive products on which the variable objects depend. INPs moreover go along with an account of modals and conditionals based on truthmaking situations.

Finally, variable objects need to be sharply distinguished from intentional objects, the nonexistent objects of thought. Intentional objects are not variable objects with possibly different manifestations in different circumstances. The reason is that intentional objects depend on cognitive products that fail to have truthmakers, such as imaginations, descriptions, and thoughts.¹⁹

References

- Beyssade, C., and C. Sorin (2005). "A Syntax-Based Analysis of Predication." *Proceedings of Semantics and Linguistic Theory* 15, 44–61.
- Bhatt, R. (2002). "The Raising Analysis of Relative Clauses: Evidence from Adjectival Modification." *Natural Language Semantics* 10: 43–90.
- Campbell, K. (1990). *Abstract Particulars*. Oxford: Blackwell.
- Fara, D. G. (2013). "Specifying Desires." *Noûs* 47(2): 250–72.
- Fine, K. (1982). "The Problem of Non-existents. I. Internalism." *Topoi* 1: 97–140.
- Fine, K. (1985). *Reasoning with Arbitrary Objects*. Oxford: Blackwell.
- Fine, K. (1999). "Things and Their Parts." *Midwest Studies of Philosophy* 23: 61–74.
- Fine, K. (2001). "The Question of Realism." *Philosophers Imprint* 1(1).
- Fine, K. (2012). "Counterfactuals without Possible Worlds." *Journal of Philosophy* 109(3): 221–46.
- Fine, K. (2017). "Truthmaker Semantics." In B. Hale and C. Wright (eds.), *Blackwell Philosophy of Language Handbook*. New York, online.
- Forbes, G. (1985). *The Metaphysics of Modality*. Oxford: Clarendon Press.

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- Grosu, A., and M. Krifka (2007). "The Gifted Mathematician that you Claim to be." *Linguistics and Philosophy* 30: 445–85.
- Grosu, A., and F. Landman (1998). "Strange Relatives of the Third Kind." *Natural Language Semantics* 6: 125–70.
- Harves, S., and R. Kayne (2012). "Having *need* and needing *have*." *Linguistic Inquiry* 43(1): 120–32.
- Koslicki, K. (2008). *The Structure of Objects*. Oxford: Oxford University Press.
- Kratzer, A. (2007/2014). "Situations in Natural Language Semantics." *Stanford Encyclopedia of Philosophy*. Online.
- Laserson, P. (1996). "Adnominal Conditionals." In T. Galloway and J. Spence (eds.), *Proceedings from Semantics and Linguistic Theory VI*. Ithaca, NY: CLC Publications, 154–66.
- Lowe, J. (2006). *The Four-Category Ontology. A Metaphysical Foundation for Natural Science*. Oxford: Oxford University Press.
- Mertz, D. W. (1996). *Moderate Realism and Its Logic*. New Haven: Yale University Press.
- Moltmann, F. (1997). "Intensional Verbs and Quantifiers." *Natural Language Semantics* 5(1): 1–52.
- Moltmann, F. (2007). "Events, Tropes and Truthmaking." *Philosophical Studies* 134: 363–403.
- Moltmann, F. (2008). "Intensional Verbs and their Intentional Objects." *Natural Language Semantics* 16(3): 239–70.
- Moltmann, F. (2009). "Degree Structure as Trope Structure: A Trope-Based Analysis of Comparative and Positive Adjectives." *Linguistics and Philosophy* 32: 51–94.
- Moltmann, F. (2013a). "Reference to Numbers in Natural Language." *Philosophical Studies* 162(3): 499–534.
- Moltmann, F. (2013b). *Abstract Objects and the Semantics of Natural Language*. Oxford: Oxford University Press.
- Moltmann, F. (2013c). "On the Distinction between Abstract States, Concrete States, and Tropes." In C. Beyssade, A. Mari, and F. Del Prete (eds.), *Genericity*. Oxford: Oxford University Press, 292–311.
- Moltmann, F. (2013d). "Tropes, Bare Demonstratives, and Apparent Statements of Identity." *Nous* 47(2): 346–70.
- Moltmann, F. (2014). "Propositions, Attitudinal Objects, and the Distinction between Actions and Products." *Canadian Journal of Philosophy* 43(5–6): 679–701.
- Moltmann, F. (2015). "Quantification with Intentional and with Intensional Verbs." In A. Torza (ed.), *Quantifiers, Quantifiers, Quantifiers*. Dordrecht: Synthese Library, 141–68.
- Moltmann, F. (2017). "Cognitive Products and the Semantics of Attitude Verbs and Deontic Modals." In F. Moltmann and M. Textor (eds.), *Act-Based Conceptions of Propositional Content. Contemporary and Historical Perspectives*. New York: Oxford University Press, 254–90.
- Moltmann, F. (2019). "Attitudinal Objects. Their Importance for Philosophy and Natural Language Semantics." In B. Ball and C. Schuringa (eds.), *The Act and Object of Judgment*. Routledge Studies in Contemporary Philosophy. New York: Routledge, 180–201.
- Montague, R. (1973). "The Proper Treatment of Quantification in Ordinary English." In J. Hintikka et al. (eds.), *Approaches to Natural Language*. Dordrecht: Reidel.
- Rodriguez-Pereyra, G. (2005). "Why Truthmakers?" In H. Beebe and J. Dodd (eds.), *Truthmakers: The Contemporary Debate*. Oxford: Oxford University Press, 17–31.
- Strawson, P. (1959). *Individuals. An Essay in Descriptive Metaphysics*. London: Methuen.

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- Twardowski, K. (1911). “Actions and Products. Some Remarks on the Borderline of Psychology, Grammar, and Logic.” In J. Brandl and J. Wolenski (eds.), *Kazimierz Twardowski. On Actions, Products, and Other Topics in the Philosophy*. Amsterdam: Rodopi, 1999, 103–32.
- Williams, D. C. (1953). “On the Elements of Being.” *Review of Metaphysics* 7: 3–18.
- Woltersdorff, N. (1970). *On Universals*. Chicago: University of Chicago Press.