Approaches to Parts and Wholes in Semantics
Advanced course
ESSLLI 2025 Summer School
Friederike Moltmann
Handout 2

The Extensional Mereological Approach to Part-Whole Structure

Recapitulation of last time

Ontology of part-whole structure:
What is the basis for deciding about ontology and about part-whole relations in particular?
Linguistically manifest intuitions about part-whole structure; but what sorts of linguistic data are relevant?
Varzi's data: sentences with *part of*
Other explicit part-whole related expressions:
Partial, complete: reference to an abstract or conceived whole
Different kinds of complete wholes: German *voellig* vs. *vollstaendig*
Indirect function of part-whole structure:
Semantics of definite plural and mass NPs, conjunctions of definite NPs, connections between the semantics of NPs and event semantic connections
Overall conclusion
Linguistic data about part-whole structure are much richer than generally considered by mereologists.

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1. Extensional mereology

Extensional mereology, informally

- Part relation is transitive.
- Part relation is closed under sum formation (fusion): unrestricted sum formation.
- Entities that share the same proper parts are identical.

Suited for completely unstructured wholes

Formal conditions of extensional mereology

(1) a. Reflexivity: $\forall x(x < x)$

b. Transitivity : $\forall x \forall y \forall z((x < y \ \& \ y < z) \rightarrow x < z)$

c. Antisymmetry: $\forall x \forall y((x < y \ \& \ y < x) \rightarrow x = y)$

d. Extensionality : $\forall x\ z \forall x \forall y((x < z \ \& \ x \neq z \ \& \ y < z \ \& \ y \neq z) \rightarrow x = y)$

<u>Sum formation or fusion *sum*</u>

Definition based on overlap:

(2) <u>Definition of overlap</u>

$x \mathrm{O} y =_{def} \exists z(z < x \ \& \ z < y)$

(3) <u>Definition of sum formation</u>

For a nonempty set P, $\mathrm{sum}(x, P) =_{def} \forall y(y \mathrm{O} x \leftrightarrow \exists z(y \mathrm{O} z \ \& \ P(z)))$

I.e., a sum of a set P is a thing such that everything which overlaps with it also overlaps with something in P, and vice versa.

Here 'sum' denotes a relation between an entity and a sum,

Alternatively: sum as an operator applying to sets, uniqueness guaranteed by Uniqueness of Sums:

(4) <u>The axiom of uniqueness of Sums (= Extensionality)</u>

Two things composed of the same parts are identical.

$\forall P(P \neq \varnothing \rightarrow \exists! z\ \mathrm{sum}(z, P))$

Other notations:

(5) a. Binary sum: $x \oplus y =_{def} \exists z\ \mathrm{sum}(z, \{x, y\})$

b. Generalized sum: for any nonempty set P, $\oplus P =_{def} \iota z\ \mathrm{sum}(z, P)$.

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2. Application of extensional mereology

<u>Unified semantics of definite NPs using sum formation</u>

(6) a. the chairman

b. $\iota x(\mathrm{sum}(x, C)$ $(= \iota x C(x))$

(7) a. the students

b. ιxsum(x, S)

(8) a. the water

b. ιxsum(x, W)

The ι-operator 'the'

ιxP(x) is defined only if P holds of exactly one individual, and when defined, it denotes that individual.

Non-Boolean conjunction: conjunction interpreted by sum formation

Boolean conjunction: &

(9) a. John and Mary embraced.

b. The men and the women gathered.

c. The oil and the vinegar were mixed.

(10) a. $j \oplus m$ = ιxsum(x, {j, m})

b. (ιxsumx(x, M)) ⊕ (ιxsum(x, W)) = (⊕M) ⊕ (⊕W) = ιxsum(x, M ∪ W)

c. (ιxsumx(x, O)) ⊕ ιxsum(x, V)) = (⊕O) ⊕ (⊕V) = ιxsum(x, O ∪ V)

(11) a. E(j ⊕ m) (formalization of 10a)

b. G(ιxsum(x, M ∪ W)) (formalization of 10b)

c. M(ιxsum(x, O ∪ V)) (formalization of 10c)

Some issues with sum formation

Sum formation does not apply to parts of individuals to yield individuals:

(12) a. the hot dog = the sausage and the bun?

b. The sausage and bun taste similar

c. The sausage and the bun cost 4 dollar.

(13) a. The hot dog tastes similar.

b. The sausage and the bun costs 4 dollar.

But compare adjective conjunctions:

With adjective conjunctions, sum formation is applicable to single individuals

(14) The Italian flag is red, green and white.

Adjective conjunction seems to be domain-neutral:

(15) a. the blue and white *pillow*

b. the blue and white *pillows*

c. the blue and white *laundry*

Other sorts of predicate conjunctions vs. NP-conjunctions

States:

(16) a. John's being calm and nervous ≠ John's being calm and John's being nervous

b. John's being calm and John's being nervous do not go together.

c. ??? John's being calm and nervous does not go together

(17) a. John's hope to win and to become rich makes sense.

b. John's hope to win and his hope to become rich make sense.

Events:

(18) a. John's walking up and down showed his impatience / lasted for an hour.

b. John's walking up and his walking down showed his impatience / lasted for an hour.

Predicate conjunction can be used to characterize complex states or events.
NP-conjunction can only lead to pluralities of states or events.
The state of being happy and calm vs the states of being happy and being calm
The act of walking up and down vs. *the act of walking up and the act of walking down.*

Conclusion

Non-Boolean NP-conjunction interpreted by sum formation can only lead to entities in the domains of pluralities and quantities (the mass domain), not individuals (denotable by singular count NPs).

Non-Boolean predicate conjunction is interpreted in a domain-neutral way.

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3. More on extensional mereology

The lattice theoretic (algebraic) perspective (Link 1983)

The domain of pluralities and h domain of quantities form join semi-lattices

Join semilattice (L, ⊕)

For any two elements $x, y \in L$, the join $x \oplus y$ is defined, and $x \oplus y \in L$.

Join operation of a lattice meets the following conditions

(19) a. associativity $(x \oplus (y \oplus z)) = (x \oplus y) \oplus z)$

b. Commutativity $(x \oplus y) = (y \oplus x)$

c. Idempotence $(x \oplus x = x)$

Impose an additional condition :

(19) d. Unique Separation: $x < y \rightarrow \exists! z(x \oplus z = y \;\&\; \neg(x \, O \, z))$

$<$ and $\oplus$ are interdefinable:

(20) $x < y \leftrightarrow x \oplus y = y$

With Unique Separation, the join operation $\oplus$ will have the same properties as the sum operation $sum_<$ in mereology.

Complete join semilattice:

A join semilattice where every subset has a join (with bottom element (join of empty set) being removed)

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4. The formal semantics of plurals and mass nouns

<u>Extensional mereology reauires distinguishing three domains of entities with three distinct part relations:</u>

The domain of <u>individuals</u> I, ordered by $<$ (part of individuals)

The domain of <u>pluralities</u> P, ordered by $<_I$ (parts of pluralities)

The domain of <u>quantities</u> (or portions or stuff) M, ordered by $<_m$ (parts of quantities)

$(P, \oplus_i)$ is a join semilattice i.e., $(P, <_i)$ is an extensional mereology

$(M, \oplus_m)$ is a join semilattice, i.e., $(M, <_m)$ is an extensional mereology

Important:

Necessity to distinguish two part relations $<_i$ and $<_m$, in addition to the part relation $<$ applying to parts of individuals.

Example:

A part of something described as a 'sum' is neither a part in the sense of $<_i$ nor in the sense of $<_m$, but only in the sense of $<$.
A part of a plurality is never a part in the sense of $<$ or $<_m$.
Individuals are atoms with respect to $<_i$, but not with respect to $<_m$, and
$<_m$ does not apply to pluralities or individuals (except in Link 1983).

The semantics of singular count, plural, mass nouns

(21) For a singular count noun extension N,
for all x, N(x), x is an atom with respect to $<_i$.

(22) $\text{atom}(x) =_{def} \neg\exists y(y < x \ \& \ x \neq y)$

Potential problems: sequence-type nouns

Continuous parts of sequences are again sequences
Similarly for sums, entities, quantities, fences, walls, Russian dolls

A problem considered a major challenge to the notion of an individual (and for the extensional mereological account of the mass-count distinction): see S. Rothstein (2017): *Semantics of Counting and Measuring*.
However, sequence-type nouns do not actually pose a problem:
A part of a sequence is not a part in the sense of $<_i$, but only in the sense of $<$, since a sequence is not a proper plurality.
A sequence and its parts are atoms – with respect to <i!

(23) The semantics of plurals within extensional mereology
For a plural noun extension N_{pl}, $N_{pl} = \oplus N$

Exclusive and inclusive conceptions of pluralities

Do pluralities include pluralities of one? Good evidence that they do.

(24) Does John have children? Yes, he has one daughter.

Plural extensions are cumulative and atomic

(25) $(N_{pl}, \oplus_i)$ is a complete join semilattice

(26) The semantics of mass nouns in extensional mereology

For mass noun extension N, (N,$\oplus_m$) is a complete join semilattice.

Mass nouns extensions are cumulative, but not atomic.

Are mass noun extensions divisive?

(27) A set N is divisive $=_{def} \forall x \exists y(y < x \;\&\; y \neq x)$

Problems

Minimal parts problem: proper parts of H_2O molecules are no longer water.

Object mass nouns:

furniture, luggage, glassware, clothing, faculty, police force

Minimal pairs:

clothes – clothing, policemen – police force, cows – cattle, pâtes – pasta

Mass noun extensions are merely perceived as being divisive?

Mass nouns extensions are merely realizations of a type that is specified as homogenous, without having to be homogenous themselves (Liebesmann 'Partialhood', 2024).

General difficulties for extensional mereology

Can the notion of an atom account really for countability, for being a single object?

In extensional mereology being an atom means: being denoted by a singular count.

Extensional mereology with its distinction into plural-specific and mass-specific domains involves strictly language-dependent part structure, dependent on the use of plural or mass categories.

This means it does not really provide a content of the morphosyntactic mass-count distinction or it does not appeal to a language-independent notion of being 'one', a single object.

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5. Arguments for a single part relation (Moltmann 1997, 1998)

1. Uniform semantics of adjective conjunction

(28) Whatever is in the bag is white and red.

2. Special quantifiers across domains

Special quantifiers: *something, everything, what, that* (see e.g., Moltmann 2022)

Special quantifiers are domain-neutral:

(29) a. *What* is in the bag? An apple, some bread, some coins

b. John ate *something*, bread, an apple, or some cookies.

c. John ate something the bread, the apples, the ten cookies.

Application of part-related expressions to special quantifiers:

(30) John ate part of what was in the bag (part of an apple, part of some bread, part of some cookies)

3. Modifiers with domain-independent part structure-related content

Part structure-sensitive adjectives

(31) a. the frequent rain

b. the frequent rainfalls

Meaning of *frequent* involves temporal separation of parts, in the generalized sense of 'part'.

4. Domain-independent part quantifiers across domains: *whole* across languages

French:

(32) a. toute la journée 'the whole day'

b. tous les jours 'the whole days'

c. tout le vin 'the whole wine'

German:

(33) a. der ganze Tag 'the whole day'

b. die ganzen Tage 'the whole days'

c. der ganze Wein 'the whole wine'

5. The noun *part* as a domain-neutral expression:

(34) a. John ate part of the apple / the fruit / the grapes

b. John ate the apple / the fruit / the grapes only in part.

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6. Extensional mereology and event semantics

Verbs have cumulative extensions.

Different thematic relations require cumulativity:

(35) a. John laughed.

b. The children laughed

(36) a. John drank the wine in the first glass.

b. John drank the wine in the first and the second glass.

Measuring out events:

(37) a. John drank the wine in the two classes in ten minutes.

b. or two hours John drank wine.

Events may inherit part structure from event participants.

Multidimensional part structures may result:

(38) The guests drank the two bottles of wine.

However, the verbal domain of events does not seem to divide into singular count-plural-mass semantically.

Rather it acts like a classifier system - roughly like nouns in Chinese.

Verbs classify as mass with respect to the Davidsonian event argument position.

E. g., numerals require the classifier *times* when applying to any verb or VP (Moltmann 1997, to appear):

(39) a. * John fell three.

b. John fell three times.

(40) John worked out three times today.

Some suggested readings

Link, G. (1983): 'The Logical Analysis of Plurals and Mass Nouns'. In R. Baeuerle et al. (eds.): *Semantics from Different Points of View*. Springer, Berlin, 302-323.

Varzi, A. (2016): 'Mereology'. *Stanford Encyclopedia of Philosophy*. online

Cotnoir, A. and A. Varzi (2021): *Mereology*, Oxford UP, Oxford.

Moltmann, F. (to appear): 'The Mass-Count Distinction, the Verbal Domain of Events, and other Non-Nominal Domains'. To appear in Lieven Dankaert, V. Mostrov and F. Talayati (ed.): *Possession and the Part-Whole Relation in Natural Language*. Oxford UP, Oxford.

Moltmann, F. (2022): 'Names, Light Nouns, and Countability'. *Linguistic Inquiry* 54(1), 117-146.

Moltmann, F. (2016): 'Plural Reference and Reference to a Plurality. Linguistic Facts and Semantic Analyses'. In M. Carrara / A. Arapinis / F. Moltmann (eds.): *Unity and Plurality. Logic, Philosophy, and Semantics*. Oxford University Press, Oxford, 93-120.

Moltmann, F. (1998): 'Part Structures, Integrity and the Mass-Count Distinction'. *Synthese* 116, 75-111

Rothstein, S. (2017): *Semantics for Counting and Measuring*. Cambridge UP, Cambridge.