*The Ontology and Semantics of Parts and Wholes*

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Fall 2024

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Handout 2

**The Extensional Mereological Approach to Part-Whole Structure**

**Recapitulation of last time**

Ontology of part-whole structure: what is the basis for deciding about ontology and about part-whole relations in particular?

Linguistically manifest intuitions about part-whole structure: but what sorts of linguistic data are relevant?

Varzi: *part of*

Other explicit part-whole related expressions:

*All of,* *most of* etc.

*Partial, complete*: reference to an abstract whole / conceived whole

Different kinds of complete wholes: German *voellig* vs. *vollstaendig*

Indirect function of part-whole structure:

Semantics of definite plural and mass NPs, conjunctions of definite NPs, connections between semantics of NPs and event semantic connections

Overall conclusion

Linguistic data about part-whole structure are richer than generally considered by mereologists.

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**1. Extensional mereology**

Extensional mereology:

Unstructured parts and wholes

Transitive part relation

Entities that share the same parts are identical

The most important assumptions about extensional mereology relevant for semantics

Axioms of extensional mereology

(1) a. Reflexivity: ∀x(x < x)

 b. Transitivity : ∀x∀y∀z((x < y & y < z) 🡪 x < z)

 c. Antisymmetry: ∀x∀y((x < y & y < x) 🡪 x = y)

Sum formation or fusion *sum*,

Definition based on overlap:

(2) Definition of overlap

 x Ο y =def ∃z(z < x & z < y)

(3) Definition of sum formation

 For a nonempty set P, sum(x, P) =def ∀y(y Ο x 🡨🡪 ∃z(y Ο z & P(z)))

A sum of a set P is a thing such that everything which overlaps with it also overlaps with something in P, and vice versa.

Here ‘sum’ as a relation between an entity and a sum,

Alternatively: sum as an operator applying to sets, given Uniqueness of Sums

The axiom of the Uniqueness of Sums (extensionality)

Two things composed of the same parts are identical.

(4) Uniqueness of sums: ∀P(P ≠ ∅ 🡪 ∃!z sum(z, P))

Other notations:

(5) a. Binary sum: x ⊕y =def ∃z sum(z, {x, y})

 b. Generalized sum: for any nonempty set P, ⊕P =def ιz sum(z, P).

(6) The ι-operator ‘the’

 ιxP(x) is defined only if P holds of exactly one individual, and when defined, it denotes

 that individual.

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**2. Application of extensional mereology**

Unified semantics of definite NPs using sum formation

(7) a. the chairman

 b. ιx(um(x, C) (= ιxC(x))

(8) a. the students

 b. ιxsum(x, S)

(9) a. the water

 b. ιxsum(x, W)

Non-Boolean conjunction: conjunction interpreted by sum formation

Boolean conjunction: &

(10) a. John and Mary embraced.

 b. The men and the women gathered.

 c. The oil and the vinegar were mixed.

(11) a. j ⊕ m = ιxsum(x, {j, m})

 b. (ιxsumx(x, M)) ⊕ (ιxsum(x, W)) = (⊕M) ⊕ (⊕W) = ιxsum(x, M ∪ W)

 c. (ιxsumx(x, O)) ⊕ ιxsum(x, V)) = (⊕O) ⊕ (⊕V) = ιxsum(x, O ∪ V)

(12) a. E(j ⊕ m)

 b. G(ιxsum(x, M ∪ W))

 c. M(ιxsum(x, O ∪ V))

Some issues with sum formation

Does not apply to parts of individuals to yield individuals:

(13) a. the hot dog = the sausage and the bun?

 b. The sausage and bun taste similar / ?? costs 4 dollar.

 c. The hot dog tastes similar.

But contrast with adjective conjunctions:

here sum formation is applicable to single individuals

(14) The Italian flag is red, green and white.

Adjective conjunction seems to be domain-neutral:

(15) a. the blue and while pillow

 b. the blue and white pillows

 c. the blue and white laundry

Other sorts of predicate conjunction

States:

(16) a. John’s being calm and nervous ≠ John’s being calm and John’s being nervous

 b. John’s being calm and John’s being nervous do not go together.

 c. ??? John’s being calm and nervous does not go together

(17) a. John’s hope to win and to become rich makes sense.

 b. John’s hope to win and his hope to become rich make sense.

Conclusion

Sum formation strictly yield entities in the domains of pluralities and quantities, not individuals (denotable by singular count NPs).

Example with events from Champollion / Krifka (p. 533):

Suppose

that there are three events e1; e2; e3 in which Al dug a hole, Bill

inserted a rosebush in it, and Carl covered the rosebush with soil. Then

one can say that there is also an event e4 in which Al, Bill, and Carl

planted a rosebush. Do we consider e4 equal to the proper sum event

e1   e2   e3? If we do, this scenario is a counterexample to the cumulativity

assumption (Kratzer, 2003). The themes of e1; e2; e3 are the hole,

the rosebush, and the soil, and the theme of e4 is just the rosebush. The

theme of e4 is not the sum of the themes of e1, e2, and e3, violating

cumulativity. A possible objection is that e4 is not actually the sum of

e1, e2, and e3. Even though the existence of e4 can be traced back to

the occurrence of e1, e2, and e3, nothing forces us to assume that these

three events are mereological parts of e4

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**3. More on extensional mereology**

The lattice theoretic (algebraic)- perspective (Link 1983)

The domain of pluralities and h domain of quantities form join semi-lattices

Join semilattice (L, ⊕)

For any two elements x, y ∈ L, the join x ⊕ y is defined, and x ⊕ y ∈ L.

Join operation of a lattice meets the following conditions

(18) a. associativity (x ⊕ (y ⊕ z)) = (x ⊕ y) ⊕ z)

 b. Commutativity (x ⊕ y) = (y ⊕ x)

 c. Idempotence (x ⊕ x = x)

Impose an additional condition :

(19) Unique Separation: x < y 🡪 ∃!z(x ⊕ z = y & ¬(x Ο z))

< and ⊕ are interdefinable:

(20) x < y 🡨🡪 x ⊕ y = y

With Unique Separation, the join operation will have the same properties as the sum operation ⊕I n mereology: reflexivity, transitivity, and antisymmetry.

Complete join semilattice: a join semilattice where every subset has a join (with bottom element, join of empty set, removed)

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**4. The formal semantics of plurals and mass nouns**

Distinguish three domains

The domain of individuals I

The domain of pluralities P

The domain of quantities (or portions or stuff) M

(P, ⊕i) is a join semilattice i.e., (P, <i) is an extensional mereology

(M, ⊕m) is a join semilattice, i.e., (M, <m) is an extensional mereology

Important:

Necessity to distinguish two part relations <i and <m, in addition to the part relation < applying to parts of individuals.

Example:

A part of something described as a ‘sum’ is neither a part in the sense of <i nor in the sense of <m, but only in the sense of <.

A part of a plurality is never a part in the sense of < or <m.

Individuals are atoms with respect to <i, but not with respect to <m, and

<m does not apply to pluralities or individuals (except in Link 1983).

The semantics of singular count, plural, mass nouns

(21) For a singular count noun extension N:

 for all x, N(x), x is an atom with respect to <i.

(22) atom(x) =def  ¬∃y(y < x & x ≠ y)

Potential problems

Sequence-type nouns:

Continuous parts of sequences are again sequences

Similarly for sums, entities, quantities, fences, walls, Russian dolls

However, these nouns do not actually pose a problem:

A part of a sequence is not a part in the sense of <i, but only in the sense of <, since a sequence is not a proper plurality.

A sequence and its parts thus are atoms – with respect to <i!

(23) For a plural noun extension Npl, Npl = ⊕N

Exclusive and inclusive conceptions of pluralities:

do pluralities include pluralities of one? Good evidence that they do.

Plural extensions are cumulative and atomic.

(24) (Npl, ⊕i) is a complete join semilattice

(24) For mass noun extension N,

 (N,⊕m) is a complete join semilattice.

Mass nouns extensions are cumulative, but not atomic.

Are mass noun extensions divisive?

(25) A set N is divisive =def ∀x∃y(y < x & y ≠ x)

Problems

Minimal parts problem: proper parts of H2O molecules are no longer water.

Object mass nouns:

*furniture, luggage, glassware, clothing, faculty, police force*

Minimal pairs:

*clothes – clothing, policemen – police force, cows – cattle, pâtes – pasta*

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**5. Arguments for a single part relation (Moltmann 1997, 1998)**

1. Uniform semantics of conjunction, in particular adjective conjunction

2. Special quantifiers across domains

(26) a. What is in the bag? An apple, some bread, some coins

 b. John ate something, bread, an apple, or some peas.

3. Modifiers with domain-independent part structure-related content

Part structure-sensitive adjectives

(27) a. the frequent rain

 b. the frequent rainfalls

temporal separation of parts, in the generalized sense of ‘part’

4. Domain-independent part quantifiers across domains: *whole* across languages

French:

(28) a. toute la journée ‘the whole day’

 b. tous les jours ‘the whole days’

 c. tout le vin ‘the whole wine‘

German:

(29) a. der ganze Tag ‘the whole day‘

 b. die ganzen Tage ’the whole days‘

 c. der ganze Wein ‘the whole wine‘

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**6. Extensional mereology and event semantics**

Verbs have cumulative extensions.

Different thematic relations require cumulativity:

(30) a. John laughed.

 b. The children laughed

(31) a. John drank the wine in the first glass.

 b. John drank the wine in the first and the second glass.

Measuring out events:

(32) a. John drank the wine in the two classes in ten minutes.

 b. or two hours John drank wine.

Events may inherit part structure from event participants.

Multidimensional part structures may result:

(33) The guests drank the two bottles of wine.

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**7. A general worry**

1. Does the notion of atom account for countability, for being a single object?

2. Extensional mereology with its distinction into plural-specific and mass-specific domains involves strictly language-dependent part structure, dependent on the use of plural or mass categories.

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**Reading**

Link, G. (1983): ‘The Logical Analysis of Plurals and Mass Terms: A Lattice-theoretical

 Approach’. In R. Bäuerle et al. (eds): *Meaning, Use and Interpretation of Language*. De

 Gruyter, Berlin, 303–323.

Champollion, L. and M. Krifka (2017): *Mereology*. In P. Dekker and M. Aloni (eds):

 *Cambridge Handbook of Semantics*. Cambridge UP, Cambridge.